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Decoding Performance of PLS codes for O3K

- We already presented O3K channel model as

Channel model: $y = N(\mu_i, \sigma_i^2), \quad i = 0 \text{ or } 1, \ (1)$

where $N(\mu_i, \sigma_i^2)$ is a Gaussian random variable with mean μ_i and variance σ_i^2, and

$$\mu_0 = 0 \qquad \sigma_0^2 = \frac{T_s \sigma_t^2}{2q^2 M^2} \qquad \mu_1 = \rho n_s \qquad \sigma_1^2 = \sigma_0^2 + F\rho n_s$$

- We also presented channel model as (which CNES also used this model in their presentations)

$$\mu_0 = 0 \qquad \sigma_0^2 = \frac{\sigma_t^2}{2T_s R_{res}^2 M^2} \qquad \mu_1 = 2\rho P_s \qquad \sigma_1^2 = \sigma_0^2 + F'\rho 2P_s \qquad F' = \frac{qF}{T_s R_{res}}$$

CNES used an APD for 10 Gbps channel with parameters: $R_{res} = 0.8, \sigma_t = 4\times10^{-11}$, *M=8, F=5*

Decoding Performance of PLS codes for O3K
Soft decision

- For AC coupled APD the channel model after APD is

Channel model: $y = N(\mu_i, \sigma_i^2), \quad i = 0 \text{ or } 1, \ (2)$

where $N(\mu_i, \sigma_i^2)$ is a Gaussian random variable with mean μ_i and variance σ_i^2, and

$$\mu_0 = -\rho P_s \qquad \mu_1 = \rho P_s$$

$$\sigma_0^2 = \frac{\sigma_t^2}{2T_s R_{res}^2 M^2} \qquad \sigma_1^2 = \sigma_0^2 + F'\rho 2P_s \qquad F' = \frac{qF}{T_s R_{res}}$$

CNES used an APD for 10 Gbps channel with parameters: $R_{res} = 0.8, \sigma_t = 4\times10^{-11}$, *M=8, F=5*

- The ML soft decoding maximizes for j=1,..,8, $x_{j,k(i)} = -\rho P_s$ if i=0 and $x_{j,k(i)} = \rho P_s$ if i=1

$$P(\boldsymbol{y}|\boldsymbol{x}_j) = \prod \frac{1}{\sqrt{2\pi\sigma_{k(i)}^2}} \exp(-\frac{1}{\sigma_{k(i)}^2}(y_k - x_{j,k(i)})^2)$$

ML Soft decision

- If there are almost equal number of 0s and 1s, we can ignore $1/\sqrt{2\pi\sigma_{k(i)}^2}$ and approximate ML soft decision rule as

Choose codeword j if the weighted correlation below is maximum where s_k =+1 or -1

$$\sum_{k=1}^{L}\left[\rho_k y_k \frac{s_{j,k}}{\sigma_0^2+\left(\frac{s_{j,k}+1}{2}\right)F'\rho_k 2P_s}\right] \qquad (3)$$

ML hard decision

- For hard decision first for each received observation y we make a decision if +1 or -1 bit transmitted denoted by y_h. Choose y_h =+1 if $$\log\frac{\sigma_0}{\sigma_1}+\frac{(y+\rho P_s)^2}{2\sigma_0^2}-\frac{(y-\rho P_s)^2}{2\sigma_1^2}>0$$
- Or if we want uncoded bit errors to choose +1 or -1 be equal then choose y_h =+1 if $y > \rho_k P_s\left(\frac{\sigma_0-\sigma_1}{\sigma_0+\sigma_1}\right)$
- A suboptimum decision is $y>0$
  otherwise in all above decisions choose y_h =-1.
- The ML hard decision rule is choose codeword j if the correlation below is maximum where $s_{j,k}$ =+1 or -1

$$\sum_{k=1}^{L} y_k s_{j,k} \qquad (4)$$

Proposed CNES PL signaling

- The 8 sequences proposed by CNES in CCSDS 142.0-B-1 Pink Book, date February 2021, for 8 modes PL signaling can be generated by the following Matlab program

```
b1(1,:)=[0 0 0 0 0 0 0 0 0 0 1];
b2(1,:)=[0 0 0 0 0 0 0 1 0 1 0];
b2(2,:)=[0 0 0 0 0 0 0 0 1 0 1];
b2(3,:)=[0 0 0 0 0 0 0 0 0 1 0];
b2(4,:)=[0 0 0 0 0 0 0 0 0 0 1];
b2(5,:)=[1 0 0 0 0 0 0 0 0 0 0];
b2(6,:)=[0 1 0 0 0 0 0 0 0 0 0];
b2(7,:)=[0 0 1 0 0 0 0 0 0 0 0];
b2(8,:)=[1 0 0 1 0 0 0 0 0 0 0];
for i=1:8
y(i,:)=zeros(2048,1);
goldseq = comm.GoldSequence('FirstPolynomial','x^11+x^2+1',...
   'SecondPolynomial','x^11+x^8+x^5+x^2+1',...
   'FirstInitialConditions',b1(i,:),...
   'SecondInitialConditions',b2(i,:),...
   'Index',4,'SamplesPerFrame',2047);
yy = goldseq();
y(i,:)=[yy;
      1];
y(i,:)
end
```

- The minimum Hamming distance between any pair of sequences is 1024. Having 1 or 0 at the end of each sequence does not change the minimum distance. We can use 0 at the end to balance the ones and zeros.
- All Gold sequences proposed by CNES having 1 at the end. Thus number of ones=number of zeros+2
- Decoding both soft and hard can be done with correlating the received noisy sequence with selected M sequences and choosing the maximum correlation. M=8 is the number of modes selected by standard.

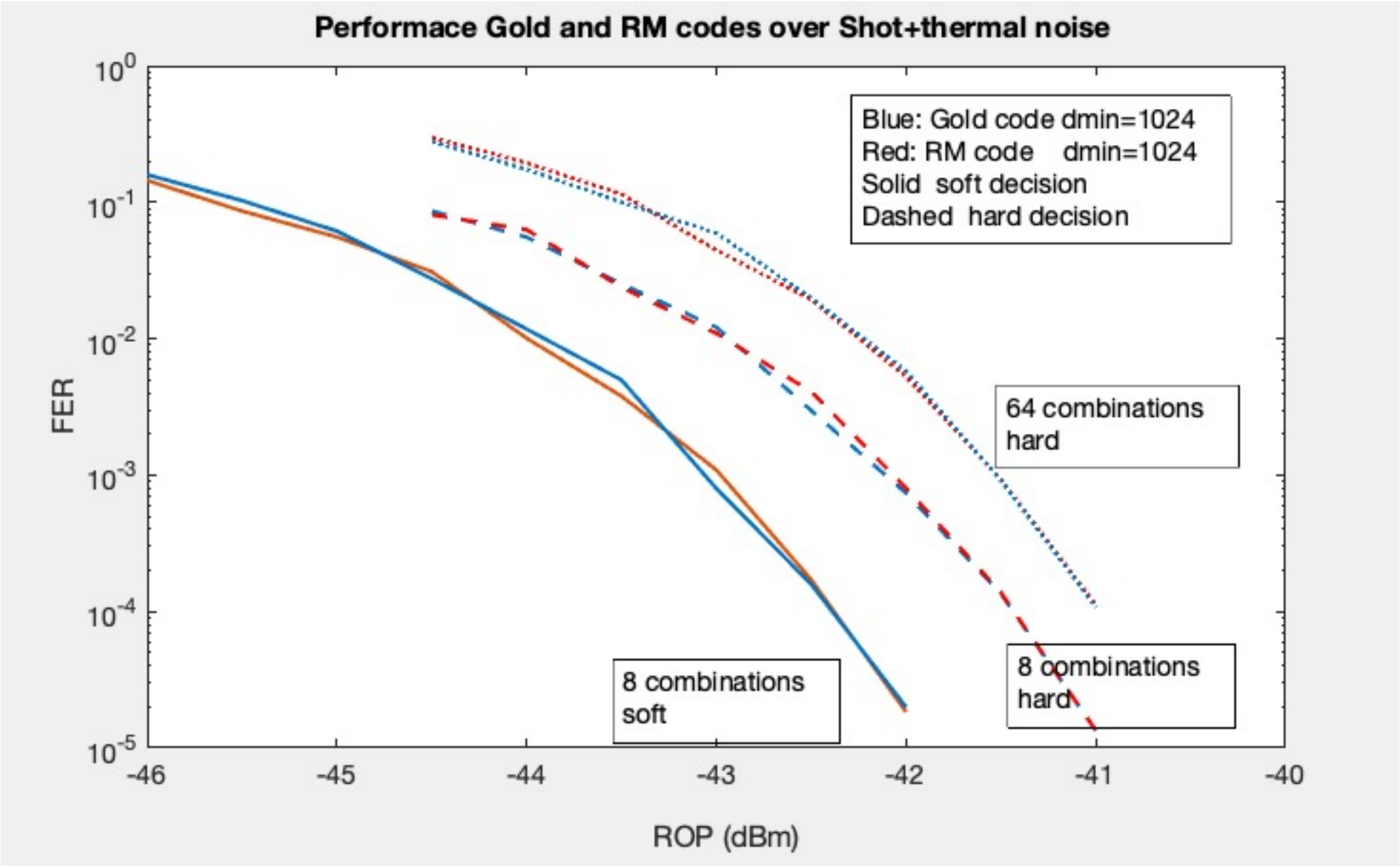
Using RM (32,6) code with 64 repetition for PL signaling modes

- Let $\boldsymbol{b}$ to represent information block of length 6 bits.
- Let $\boldsymbol{x}$ to represent codeword block of length 32 bits.
- Then $\boldsymbol{x} = \boldsymbol{b}G$ where G is 6x32 generator matrix given by

$$G = \begin{bmatrix} 0&1&0&1&0&1&0&1&0&1&0&1&0&1&0&1&0&1&0&1&0&1&0&1&0&1&0&1&0&1&0&1 \\ 0&0&1&1&0&0&1&1&0&0&1&1&0&0&1&1&0&0&1&1&0&0&1&1&0&0&1&1&0&0&1&1 \\ 0&0&0&0&1&1&1&1&0&0&0&0&1&1&1&1&0&0&0&0&1&1&1&1&0&0&0&0&1&1&1&1 \\ 0&0&0&0&0&0&0&0&1&1&1&1&1&1&1&1&0&0&0&0&0&0&0&0&1&1&1&1&1&1&1&1 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1 \\ 1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1 \end{bmatrix}$$

- Let $x_1 = b_1 G$ and $x_2 = b_2 G$ where $b_1 \neq b_2$ then the Hamming distance between x_1 and x_2 can be computed as weight of $x_1 + x_2$ since $x_1 + x_2 = (b_1 + b_2)G$ using addition operation over GF(2)
- However since $b_1 \neq b_2$ then $b = b_1 + b_2 \neq 0$ we can can generate 63 distinct pairs since there are 63 distinct nonzero values for b with corresponding codewords $x_1$and x_2 with Hamming distance at least 16 between them. There is only one b=[000001] that results in Hamming distance of 32.
- If each distinct nonzero input bit sequence represent a mode then there are 63 nonzero codewords with weight of at least 16. We can add a non-codeword to all codewords to have 64 nonzero sequences.
- Now if we repeat the same codeword 64 times we generate a sequence of length 2048 with Hamming distance between any two distinct sequences is at least d=1024. So we can support 64 modes for PL signaling.
- Decoding both soft and hard can be done with correlating the received noisy sequence with the selected M sequences and choosing the maximum correlation. 8<=M<64 is the number of available modes.

Using soft and hard decision



Is it possible for 8 mods to have better code?

- There are many bounds on minimum distance, W e show a simple one here.
- A simplex code for block length $n = 2^k - 1$ with rate k/n has $d_{min} = 2^{k-1}$
- In general we can show $\frac{d_{min}}{n} \leq \frac{2^{k-1}}{2^k - 1} + \epsilon$. If we ignore ϵ then for 8 codewords and n=2048 we roughly get minimum distance 1170.
- Here for 8 codewords we found a new code (32,3) with Generator matrix (we can add a non-codeword sequence to all codewords to avoid an all zero sequence)

$$G = \begin{bmatrix} 1\,0\,0\,1\,1\,0\,1\,0\,1\,1\,0\,0\,1\,1\,1\,1\,0\,0\,1\,1\,0\,1\,0\,1\,0\,1\,1\,0\,1\,0\,0\,1 \\ 0\,1\,0\,1\,0\,1\,1\,0\,1\,0\,0\,1\,1\,0\,1\,0\,1\,1\,0\,0\,1\,1\,1\,1\,0\,0\,1\,1\,0\,1\,0\,1 \\ 0\,0\,1\,1\,0\,1\,0\,1\,0\,1\,1\,0\,1\,0\,0\,1\,1\,0\,1\,0\,1\,1\,0\,0\,1\,1\,1\,1\,0\,0\,1\,1 \end{bmatrix}$$

and minimum distance 18. I we repeat the codewords 64 times then 64x18= 1152.

Performance of the new code compared with Gold and RM we repetition using soft and hard decision