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PN ranging-code schemes--past & future

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Content of this presentation:

- Development of a simple metric, **normalized acquisition time**, for comparing ranging sequences.
- Determination of the normalized acquisition time for the **1999 JPL ranging-sequence scheme**.
- Formulation of the "**weighted-voting Titsworth ranging-sequence scheme**" as a clear improvement.
- Formulation of the "**weighted-voting Stiffler ranging-sequence scheme**" as a further improvement except for its spectral properties.
- Formulation of a "**scrambled weighted-voting Stiffler ranging-sequence scheme**" with same acquisition time, but excellent spectral properties
- Specification of an **effective acquisition algorithm** for the "scrambled weighted-voting Stiffler scheme".

Probing Sequences

In all practical ranging systems, the **±1 periodic ranging sequence** is acquired by the receiver as the result of **correlations** between the received sequence and **certain ±1 periodic sequences** that we will call **probing sequences**.

For each probing sequence, one or more correlations are made to determine that shift of this probing sequence that is "**in-phase**" with the received sequence.

The probing sequences may be subsequences of the ranging sequence or they may be related to the ranging sequence in less direct ways, e.g., the ranging sequence might be the result of some kind of vote among corresponding chips of the probing sequences.

The 1999 JPL Scheme

The first periods of the six "**component sequences**" for the 1999 JPL ranging-sequence scheme are:

C1 = +1 -1 (the so-called **range-clock** component)
C2 = +1 +1 +1 -1 -1 +1 -1
C3 = +1 +1 +1 -1 -1 -1 +1 -1 +1 +1 -1
C4 = +1 +1 +1 +1 -1 -1 -1 +1 -1 -1 +1 +1 -1 +1 -1
C5 = +1 +1 +1 +1 -1 +1 -1 +1 -1 -1 -1 -1 +1 +1 -1 +1 +1 -1 -1
C6 = +1 +1 +1 +1 +1 -1 +1 -1 +1 +1 -1 -1 +1 +1 -1 -1 +1 -1 +1 -1 -1 -1 -1

The ranging sequence is obtained by combining the chips of the six periodic component sequences at the same position in the manner that the result is +1 if and only if C1 has a +1 at that position or if all five of the sequences C2, C3, C4, C5 and C6 have a +1 at that position.

The **period** of the ranging sequence is
$2 \times 7 \times 11 \times 15 \times 19 \times 23 =$ **1 009 470**.

The 1999 JPL ranging-sequence scheme is described in detail in:

J. B. Berner, J. M. Layland, P. W. Kinman and J. R. Smith, "Regenerative Pseudo-Noise Ranging for Deep-Space Applications", Telecommunications and Mission Operations (TMO) Progress Report 42-137, Jet Propulsion Laboratory, Pasadena, CA, May 15, 1999.

The rule for forming the ranging sequence from the component sequences ensures that the **range-clock component C1**, which is an alternating +1/-1 sequence, **is strongly present in the ranging sequence**. This is important for locking onto the chips of the ranging sequence at the receiving site.

The **probing sequences** are the six component sequences and their distinct cyclic shifts. The number of probing sequences, excluding the range clock, is thus 7 + 11 + 15 + 19 + 23 = 75.

The correlations of the six +1/-1 **component sequences** with one period of the +1/-1 ranging sequence are as follows:

C1:   963390 -963390
C2:   46080 0 0 0 0 0 0
C3:   46080 0 0 0 0 0 0 0 0 0 0
C4:   46080 0 0 0 0 0 0 0 0 0 0 0 0 0 0
C5:   46080 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
C6:   46080 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

This means that the **probing sequence** that is the 2nd right cyclic shift of C3 gives the following correlations
0 0 46080 0 0 0 0 0 0 0 0
with one period of the ranging sequence, and so on.

Because the period of the ranging sequence is 1 009 470, we see from its correlations (± 963 390) with C1 that **the ranging sequence is very closely approximated by the range-clock sequence C1 itself**.

A detailed analysis of the 1999 JPL ranging sequence shows:

- The number of -1's in the sequence is 481 695
- The number of +1's in the sequence is 527 775
- The longest run of -1's has length 1.
- The longest run of +1's has length 7.
- The number of transitions (-1 to +1 and +1 to -1) is 963 390.

ACQUISITION OF A PROBING SEQUENCE

The "in-phase" position of probing sequence is acquired by first **correlating the received noisy ranging sequence** (starting at some unknown point), over a specified number of periods of the ranging sequence, **with all the cyclic shifts of that probing sequence**, then **using the maximum of these correlations to decide which cyclic shift is in-phase** with the signal component of the received sequence.

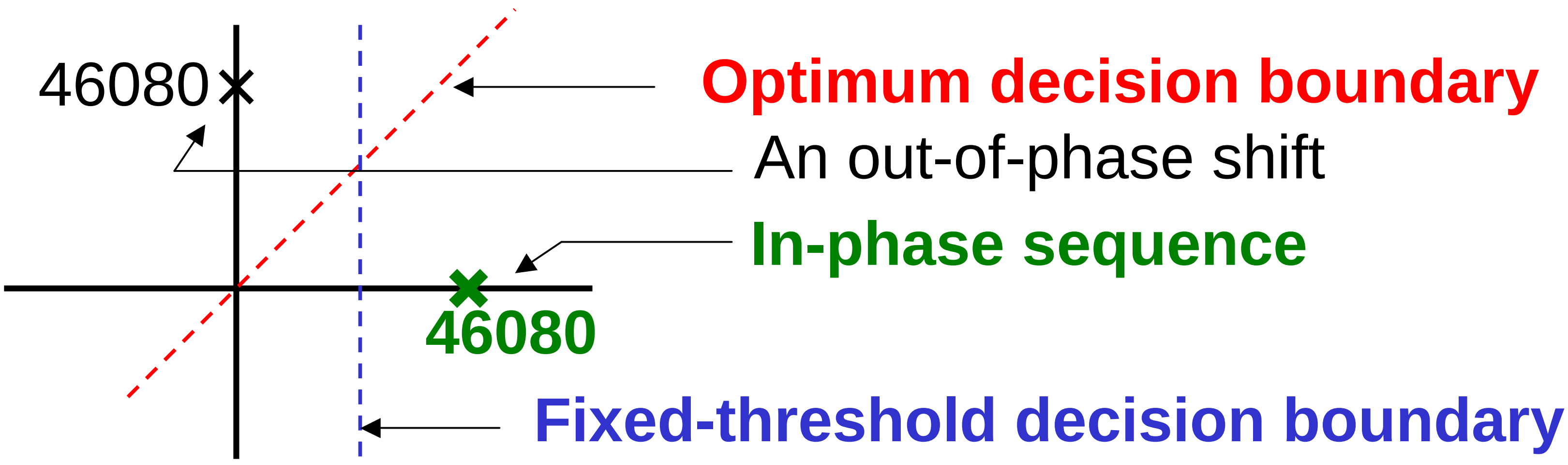
Exception: For the **alternating +1/-1 sequence (i.e., the range clock)**, **one correlation suffices** because the correlation with this sequence is just the negative of the correlation with its cyclic shift.

Under the assumption of additive white Gaussian noise, **the decision rule just stated maximizes the probability of correct acquisition** of the probing sequence. (This is the **“matched-filter receiver”** for equal-energy signals.)

Warning: Comparison to a fixed threshold when making the decision on the in-phase position for **orthogonal alternatives** incurs a **3 dB penalty**. (See next slide for reason.)

(cf. J. M. Wozencraft and I. M. Jacobs, *Principles of Communication Engineering*. New York: Wiley, 1965, pp. 233-263, for a thorough treatment of the underlying principles.)

Example (using the 1999 JPL ranging-sequence):
Consider the acquisition of the probing sequences that are the cyclic shifts of C2.



The signal points are farther from the optimum decision boundary by a factor of $\sqrt{2}$ compared to the fixed-threshold boundary.

Normalized acquisition time

(a metric for comparing ranging sequences)

The idea is to compare probing sequences to the case of antipodal +1/-1 sequences (such as the range-clock sequence C1 and its shift by one chip).

The **probability of error** in an "in-phase" decision between two **antipodal** sequences of length K chips with energy E_C per chip when the noise is additive white Gaussian noise with two-sided power spectral density $N_0/2$ is given by (cf. Wozencraft & Jacobs, p. 250)

$$P_e = Q\left(\sqrt{2KE_C / N_0}\right)$$

where

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-t^2/2} dt.$$

Equivalently, the **number *K* of chips needed** for a given P_e with **antipodal** signals is

$$K = \frac{\left[Q^{-1}(P_e)\right]^2}{2E_C / N_0}.$$

This is the key to evaluating the efficiency of probing sequences. The reason is that the **number *K* of chips needed** for a given P_e with **antipodal** signals is rather **insensitive to** the chosen value of P_e for any specified value of the signal-to-noise ratio.

P_e	K (for $2E_C/N_0$ of –30 dB)*
10^{-4}	13 947.
10^{-5}	18 281.
10^{-6}	22 672.
10^{-8}	31 551.
10^{-10}	40 512.
10^{-12}	49 521.

Range of primary interest here (values 10^{-5} to 10^{-6})

*Corresponds to a "received ranging power-to-noise ratio" P_r/N_o of 27 dBHz, as used in the 1999 JPL report, and a chip time of 10^{-6} s.

Assuming that we want P_e to be about **10^{-5}**, which we will soon see corresponds to a probability of successful acquisition of the ranging sequence of about 0.999, we see that we can assume that ***K* is about 20 000 chips**. This is the figure we will use in our examples.

We now specify the **parameters of an arbitrary probing sequence** that will allow us to determine how many chips of the **received sequence** must be correlated with this probing sequence to obtain the specified P_e for an incorrect in-phase decision for this probing sequence against one of its competing phases.

For all of the ranging-sequence schemes considered in this presentation, the probability of unsuccessful acquisition of the ranging sequence, **1 - P_{acq}**, is about 20 times the probability of an incorrect in-phase decision* for the worst probing sequence against one of its competing phases. Thus, assuming we want the probability of successful acquisition, **P_{acq}** , to be 0.999 or better, we should choose $P_e \leq (0.001)/20 = 5\times10^{-5}$, which is why we have taken $P_e \approx 10^{-5}$.

*See slides 28 and 46 for explanation.

The key is that KE_C in the expression for P_e is **proportional to the squared Euclidean distance** between the antipodal signals of length K chips. Thus, we need only determine what fraction of this squared Euclidean distance (or equivalently what fraction of the **signal-to-noise ratio**) is achieved between the probing sequence and the out-of-phase cyclic shift to which it is compared.

The two relevant parameters of a probing sequence are:

ξ = **average percentage correlation**
γ = **correlation scale factor**

(N.B. All of the probing sequences considered in this presentation are correlated with every chip of the received ranging sequence so there is <u>no interleaving factor</u> whose loss must be taken into account.)

Average percentage correlation ξ

C_{in} = in-phase correlation

n_{act} = number of chips actually correlated with the probing sequence to obtain C_{in}

$$\xi = \frac{C_{in}}{n_{act}}$$

<u>Example</u>: Recall for the 1999 JPL probing sequence C3, the correlations were

C3: 46080 0 0 0 0 0 0 0 0 0 0.

This correlation is done over all n_{act} = 1 009 470 chips in one period.

$\Rightarrow$ ξ = 46080/1009470 = **0.04565**

Because the signal power at the receiver is proportional to C_{in}^2 for the actual probing sequence but similarly proportional to n_{act}^2 for antipodal signals of the same length, the **signal-to-noise ratio** for (or, equivalently, the **squared Euclidean distance** between) the probing sequence and the out-of-phase cyclic shift to which it is compared is ξ^2 times that for antipodal sequences of the same length n_{act}.

Example: (continued) ξ^2 = **0.00208**

(loss of 26.8 dB compared to 100% correlation)

Correlation scale factor λ

The **correlation scale factor** λ is the ratio of the **squared Euclidean distance** between the **actual signals** and between **antipodal signals of the same energy**. The figure on the next slide shows that

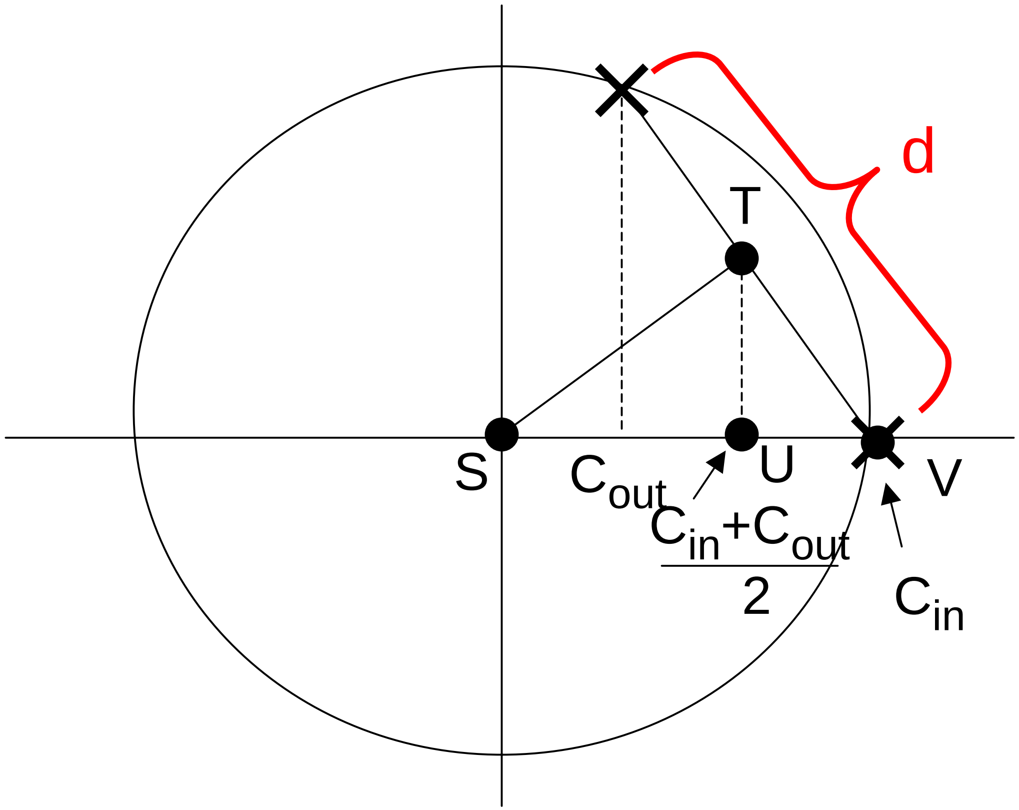
$$\lambda = \frac{C_{in} - C_{out}}{2\,C_{in}}$$

where C_{in} and C_{out} are the correlations for the in-phase probing sequence and for the competing phase, respectively.

For antipodal alternatives ($C_{out} = - C_{in}$), $\lambda = 1$

For orthogonal alternatives ($C_{out} = 0$), $\lambda = ½$ (3 dB loss)

Calculating the correlation scale factor



STV and TUV are similar triangles!

$$\Rightarrow \quad \frac{\frac{1}{2}\,d}{C_{in} - \frac{C_{in}+C_{out}}{2}} = \frac{C_{in}}{\frac{1}{2}\,d}$$

so that $d^2 = 2\,C_{in}(C_{in} - C_{out})$,

whereas for antipodal signals, $d^2 = 4\,C_{in}^2$.

Example: (continued) Recalling again that for the 1999 JPL probing sequence C3, the correlations were

C3:   46080 0 0 0 0 0 0 0 0 0 0.

Each in-phase decision is thus a decision between **orthogonal alternatives** (C_{out} = 0) so that $\lambda = ½$ (3 dB loss compared to antipodal alternatives).

Normalized correlation time

We define the **normalized correlation time,** $\boldsymbol{\tau_{cor}}$ **,** of a probing sequence with parameters ξ and λ to be

$$\tau_{cor} = \frac{1}{\lambda \xi^2}$$

Because $\frac{1}{\lambda \xi^2}$ is the factor by which the signal-to-noise ratio for the decision for a probing sequence against one of its shifts is reduced compared to antipodal signals, it follows that:

To find the correlation time for a probing sequence measured in chips of the received sequence, we just need to multiply τ_{cor} for that probing sequence by the parameter *K*, the number of such chips needed for the specified P_e with antipodal signals

<u>Example:</u> (continued) For the 1999 JPL probing sequence C3 (and indeed for all the probing sequences in this ranging scheme) except the range-clock C1, the **normalized correlation time** is

$$\tau_{cor} = \frac{1}{½ \times 0.00208} \approx 960$$

For $E_C/2N_0$ of –33 dB and $P_e \approx 10^{-5}$, $\boldsymbol{K}$ ≈ 20 000 chips.

⇒ the **correlation of C3** must be performed with at least 960 × 20 000 = **19 200 000 received chips**. This requires **19.2 s** for a chip time of 10^{-6} s.

(N.B. **Each of the six probing sequences that are shifts of C3 must also be correlated with 19 200 000 received chips before the appropriate phase of C3 is acquired**.)

Normalized acquisition time

To **acquire** a particular probing sequence, one needs in principle to correlate the received sequence with each cyclic shift of that probing sequence, then choose the cyclic shift with greatest correlation. Thus, it is natural to define the **normalized acquisition time** $\tau_{\mathbf{acq}}$ of a given probing sequence as

$$\tau_{acq} = n_s \, \tau_{corr}$$

where n_s is the number of cyclic shifts of the probing sequence that must be correlated with the received sequence.

N.B. This assumes that a single correlator will be used to perform all n_s correlations for this particular probing sequence, which is the usual case in practice.

Remark: n_s is the number of distinct cyclic shifts of the probing sequence, **except** when the probing sequence is the alternating ± 1 sequence, in which case $n_s = 1$ as either of the correlations is the negative of the other.

Example: (continued) For the 1999 JPL probing sequence C3, there are seven distinct cyclic shifts. Thus, the normalized acquisition time for this probing sequence is

$$\tau_{acq} = 7 \times \tau_{corr} \approx 7 \times 960 = \mathbf{6\,720}.$$

Example: For the 1999 JPL probing sequence C1, the range-clock sequence, n_s = 1. Thus

$$\tau_{acq} = \tau_{corr}.$$

The correlations for C1 are ± 963 390 when done over all n_{act} = 1 009 470 chips in one period. Thus ξ = 963390/1009470 = 0.95435 and ξ^2 = 0.91079. Moreover, λ = 1 (antipodal alternatives). Thus the normalized correlation time for C1 is

$$\tau_{cor} = \frac{1}{0.91079} \approx 1.10$$

which is smaller by a factor of about 875 than the correlation time for the probing sequence C3. The acquisition time for C1 is $\tau_{acq} = \tau_{corr} \approx$ 1.10, which is smaller by a factor of about 6,120 than the acquisition time for C3.

Normalized total acquisition time

We now consider the total time $\tau_{acq\text{-}tot}$ required to acquire the phase of the **entire ranging sequence**, i.e., to acquire all of the probing sequences. This parallel acquisition time will depend on the number of probing sequences that can be correlated in parallel.

•We use the 1999 JPL ranging scheme as an **example to illustrate how normalized total acquisition time is calculated**.

The assumptions are that

• the probing sequence C1 (the “range-clock component”) will first be acquired by itself.

• thereafter, the **five probing sequences C2, C3, C4, C5 and C6 will be acquired in parallel**.

• Acquisition of C1 requires a normalized time of $\tau_{acq} \approx 1.10$.

• The required normalized time for parallel acquisition of the probing sequences C2, C3, C4, C5 and C6 is equal to the largest normalized acquisition time of these five probing sequences, which is that for C6, namely $\tau_{acq} = 23 \times \tau_{corr} \approx 23 \times 960 = 22\,080$.

Thus the **normalized total acquisition time** for the 1999 JPL ranging sequence is

$$\tau_{acq\text{-}tot} \approx 1.10 + 22\,080 \approx \mathbf{22\,081.}$$

For $2E_C/N_0$ of –30 dB and $P_e \approx 10^{-5}$, $\boldsymbol{K} \approx 20\,000$ chips $\Rightarrow$ the acquisition of the entire ranging sequence requires about 441 620 000 received chips or about 442 s (7.36 min.) for a chip time of 10^{-6} s.

Probability of successful acquisition

Let P_e denote the probability of an incorrect in-phase decision for C6 against one of its competing phases. Because there are 23 - 1 = 22 incorrect phases, it follows that the probability P_{e_C6} of incorrectly acquiring probing sequence C6 is well approximated via

$$1 - P_{e\_C6} = (1 - P_e)^{22} \approx 1 - 22P_e$$

which is just the assumption that the 22 error events are independent. Thus, $\boldsymbol{P_{e\_C6} \approx 22\, P_e}$.

Assuming the correlations with C2, C3, C4 and C5 are continued over all the time that C6 is correlated, the probability of incorrectly acquiring these probing sequences is much smaller than P_{e_C6}. Thus, to a good approximation, the probability $\mathbf{1 - P_{acq}}$ of unsuccessfully acquiring the ranging sequence is

$$\mathbf{1 - P_{acq}} \approx P_{e\_C6} \approx 22\, P_e.$$

Improving the 1999 JPL ranging-sequence scheme

The **"trick"** is to take the ranging sequence to be the result of **weighted voting** of the six component sequences C1, C2, C3, C4, C5 and C6 in the manner that

- the chips of C2, C3, C4, C5 and C6 each have one vote
- but the **chips of the range-clock sequence C1 have v votes** where v can be chosen as either 2 or 4.

N.B. The possible values of v are chosen so that ties in the voting cannot occur.

We refer to this new scheme as the **weighted-voting Titsworth ranging-sequence scheme** because it differs essentially only in allowing weighted voting from the scheme in

R. C. Titsworth, "Optimal Ranging Codes", *IEEE Trans. Space Elec. & Telem*., vol. SET-10, pp. 19-30, March 1964.

v = 4 weighted-voting Titsworth ranging-sequence scheme

Combine the chips of the six periodic component sequences at each chip position by **weighted voting** with **C1** given **v = 4 votes** and the other sequences 1 vote each.

The correlations of C1, C2, C3, C4, C5 and C6 with one period (1 009 470 chips) of this +1/-1 ranging sequence are as follows:

C1: 942600 -942600

C2: 66870 -6930 -6930 -6930 -6930 -6930 -6930

C3: 66870 -4158 -4158 -4158 -4158 -4158 -4158 -4158 -4158 -4158 -4158

C4: 66870 -2970 -2970 -2970 -2970 -2970 -2970 -2970
-2970 -2970 -2970 -2970 -2970 -2970 -2970

C5: 66870 -2310 -2310 -2310 -2310 -2310 -2310 -2310 -2310 -2310 -2310
-2310 -2310 -2310 -2310 -2310 -2310 -2310 -2310

C6: 66870 -1890 -1890 -1890 -1890 -1890 -1890 -1890 -1890 -1890 -1890
-1890 -1890 -1890 -1890 -1890 -1890 -1890 -1890 -1890 -1890
-1890 -1890

The parameters of these six probing sequences (under the same assumptions as before) are

	ξ	ξ^2	λ	τ_{corr}	τ_{acq}
C1:	0.9334	0.8719	1	1.1	1.1
C2:	0.0662	0.0044	0.5518	413.0	2,891.
C3:	0.0662	0.0044	0.5311	429.1	4.720.
C4:	0.0662	0.0044	0.5222	436.4	6,546.
C5:	0.0662	0.0044	0.5173	440.6	8,371.
C6:	0.0662	0.0044	0.5141	443.3	10,195.

The **normalized total acquisition time** for this ranging sequence is

$$\tau_{acq\text{-}tot} \approx 1.1 + 10\ 195. \approx \mathbf{10\ 196.}$$

which is less than half that of the 1999 JPL ranging sequence.

Note that the range-clock component of the **v = 4 weighted-voting Titsworth ranging sequence** is virtually as strong (correlation ± **942 600**) as it was in the original 1999 JPL ranging sequence (correlation ± 963 390). The tracking loop for synchronizing the range clock will give the same small error as in the 1999 JPL ranging-sequence scheme.

The spectral properties of the **v = 4 weighted-voting Titsworth ranging sequence** are also virtually the same as for the 1999 JPL ranging sequence.

The number of -1'a in the sequence is **492 090** (481 695)
The number of +1's in the sequence is **517 380** (527 775)
The longest run of -1's has length **5** (1)
The longest run of +1's has length **7** (7)
Number of transitions (-1 to +1 and +1 to -1) is **945 480** (963 390)

There is a slightly better balance of +1's and -1's but a slightly smaller number of transitions.

v = 2 weighted-voting Titsworth ranging-sequence scheme

Combine the chips of the six periodic component sequences at each chip position by **weighted voting** with **C1** given **v = 2 votes** and the other sequences 1 vote each.

The correlations of the component +1/-1 sequences with this +1/-1 ranging sequence are

```
C1: 623400 -623400
C2: 261510  -26906 -26906 -26906 -26906 -26906 -26906
C3: 259374  -15930 -15930 -15930 -15930 -15930 -15930 -15930
            -15930 -15930 -15930
C4: 257910  -11274 -11274 -11274 -11274 -11274 -11274 -11274
            -11274 -11274 -11274 -11274 -11274 -11274 -11274
C5: 256926   -8714  -8714  -8714  -8714  -8714  -8714  -8714
             -8714  -8714  -8714  -8714  -8714  -8714  -8714
             -8714  -8714  -8714  -8714
C6: 256230   -7098  -7098  -7098  -7098  -7098  -7098  -7098
             -7098  -7098  -7098  -7098  -7098  -7098  -7098
             -7098  -7098  -7098  -7098  -7098  -7098  -7098
             -7098
```

The parameters of these six probing sequences (under the same assumptions as before) are

	ξ	ξ^2	λ	τ_{corr}	τ_{acq}
C1:	0.6176	0.3814	1	2.62	2.62
C2:	0.2591	0.0671	0.5514	27.02	189.2
C3:	0.2569	0.0660	0.5307	28.54	314.0
C4:	0.2555	0.0653	0.5219	29.36	440.3
C5:	0.2545	0.0648	0.5170	29.86	567.4
C6:	0.2538	0.0644	0.5139	30.21	694.7

The **normalized total acquisition time** for this ranging sequence is

$\tau_{acq\text{-}tot} \approx 2.6 + 694.7 \approx$ **697.**

which is about **32 times smaller** than that of the1999 JPL ranging sequence.

Note that the range-clock component of the **v = 2 weighted-voting Titsworth ranging sequence**, while strong, is nonetheless **3.6 dB smaller** (correlation ± **623 400**) than in the original 1999 JPL ranging sequence (correlation ± 963 390). The tracking loop for synchronizing the range clock will give a somewhat larger mean-squared error than in the 1999 JPL ranging-sequence scheme.

It is now a matter of engineering judgement to decide whether the reduction of acquisition time by a factor of about 32 is worth this sacrifice of increased tracking error.

The spectral properties of the **v = 2 weighted-voting Titsworth ranging sequence** are not much inferior to those of the 1999 JPL ranging sequence.

The number of -1'a in the sequence is **454 698** (481 695)

The number of +1's in the sequence is **554 772** (527 775)

The longest run of -1's has length **7** (1)

The longest run of +1's has length **11** (7)

Number of transitions (-1 to +1 and +1 to -1) is **706 200** (963 390)

Weighted-voting Stiffler ranging-sequence scheme

This is another new scheme that offers some **substantial improvements** over the previous ranging-sequence scheme but with one serious disadvantage, namely **poor spectral properties**.

For clarity, we first describe this scheme before describing a modification that retains all the advantages but that also provides excellent spectral properties.

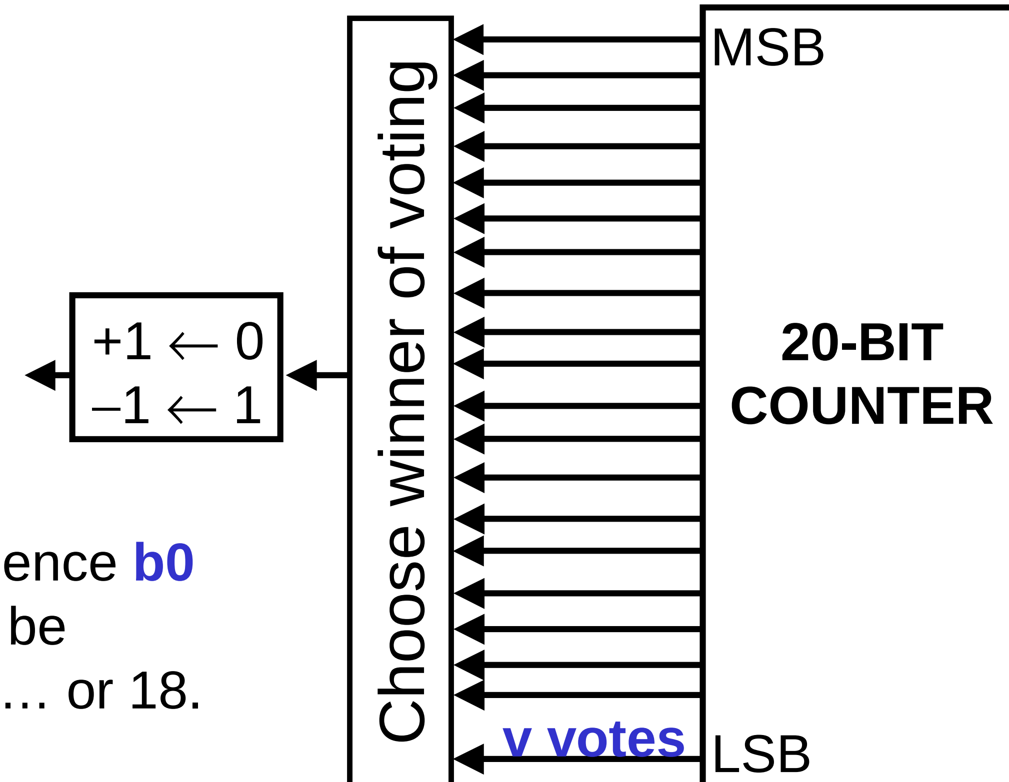
The original Stiffler scheme is described in:
J.J. Stiffler, "Rapid Acquisition Sequences," *IEEE Trans. Info. Th.*, vol. IT-14, pp. 221-225, March 1968

In what follows, we use the ESA convention that a binary 0 corresponds to a +1 chip and that a binary 1 corresponds to a -1 chip.

Weighted-voting Stiffler ranging-sequence scheme

The counter is started
in the all-zero state.

±1 ranging sequence



The chips of the LSB sequence **b0**
have **v votes** where v can be
chosen as 2 or 4 or 6 or 8 … or 18.

The least significant bit (LSB) sequence is
b0 = 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 . . .   (i.e, the "range-clock" sequence)
The next-to-least significant bit sequence is
b1 = 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1 1 . . .   and so forth.

- The **period** of the ranging sequence is 2^{20} = **1 048 576** which is convenient for locking to other carriers.

- The probing sequences are the 20 bit sequences **b0**, **b1**, **b2**, … , **b19**. As shown on the next slide, they can be acquired in this order in such a way that (1) **a single correlation is required to acquire each probing sequence**, and (2) **the decision is between antipodal alternatives** for each of the 20 probing sequences. (This was also a feature of the original Stiffler ranging-sequence scheme.)

- The nine possible values of **v** (2 or 4 or 6 or … or 18) give **flexibility** in choosing the parameters of the ranging sequence to meet system requirements (slide 43 gives corresponding values of in-phase correlation and normalized correlation time).

The idea of the acquisition algorithm can be seen by considering the first three probing sequences.

b0 = 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 . . .

b1 = 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1 1 . . .

b2 = 0 0 0 0 1 1 1 1 0 0 0 0 1 1 1 1 . . .

After correlating one or more periods of the received sequence with **b0** and finding that the first received chip corresponds to a 1 in **b0**, one knows that the received sequence is aligned with **b1** as

either 0 1 1 0 0 1 1 0 0 1 1 0 0 1 1 0 . . .

or 1 0 0 1 1 0 0 1 1 0 0 1 1 1 1 0 . . .

Thus, if one now **skips one chip** in the received sequence, the acquisition of **b1** is just the decision between these two antipodal sequences and requires just a single correlation with **b1**.

Having acquired **b1**, we can by **skipping two chips** (if necessary) acquire **b2** with one correlation, etc.

Although the 20 probing sequences **b0**, **b1**, **b2**, … , **b19** must be acquired in this order, groups of probing sequences can be acquired in parallel in the manner we now explain for parallel acquisition of **b1** and **b2**.

After acquiring **b0** and, if necessary, skipping one chip, the alignment of **b1** with the received sequence is

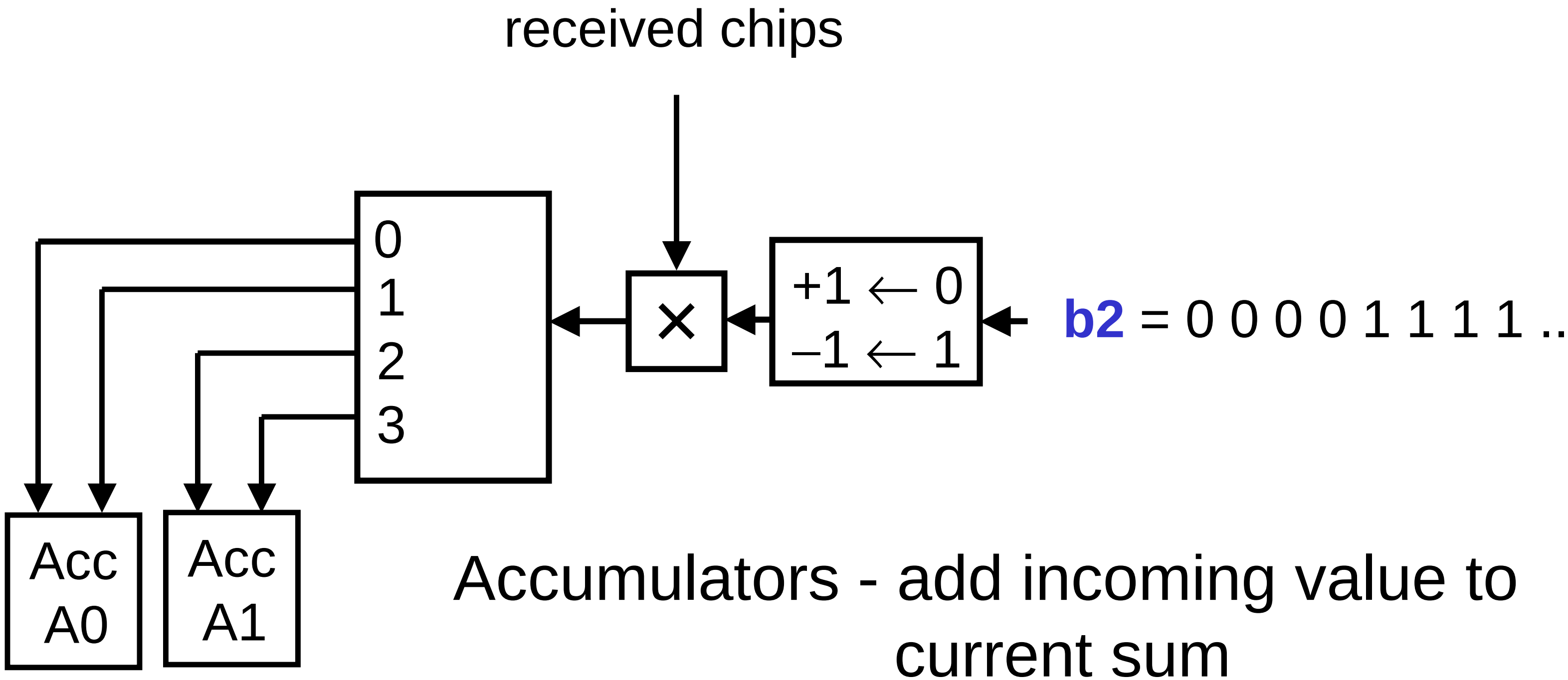
0 0 1 1 0 0 1 1 0 0 1 1 0 0 1 1 . . .

while the alignment of **b2** is

either **0 0 0 0 1 1 1 1 0 0 0 0 1 1 1 1** . . . (**b2** itself)

or **0 0 1 1 1 1 0 0 0 0 1 1 1 1 0 0** . . . (**b2** shifted)

where a red digit indicates that the corresponding received chip should be multiplied by +1 in the correlation and a green digit indicates that it should be multiplied by -1. Thus, if we temporarily store the correlations with **b2** in two separate accumulators as shown on the next slide,



then, after the decision on the phase of the probing sequence **b1** has been made,

- if the decision is that **b1** itself is in phase (so no digits are skipped) then A0 + A1 is the correlation for **b2.**
- if the decision is that **b1** has been shifted (so two digits are skipped) then A0 - A1 is the correlation for **b2.**

The previous example generalizes to any number of bit sequences to be correlated in parallel.

If m bit sequences are to be correlated in parallel, then

• for the correlation with the first sequence one requires only one accumulator

• for the correlation with the second sequence one requires two accumulators

• for the correlation with the third sequence one requires four accumulators

• for the correlation with the fourth sequence one requires eight accumulators, etc.

Weighted-voting Stiffler ranging-sequence scheme

	range clock **b0**		sequences **bi**, i > 0	
	C_{in}	τ_{acq}	C_{in}	τ_{acq}
v = 2	369512	8.0527	175032	35.9
v = 4	671840	2.4359	127296	67.9
v = 6	**873392**	**1.4414**	**74256**	**199.4**
v = 8	**981920**	**1.1404**	**34272**	**936.1**
v = 10	1028432	1.0396	12240	7339.0
v = 12	1043936	1.0089	3264	103204.7
v = 14	1047812	1.0015	612	2935600.8
v = 16	1048496	1.0002	72	212097152.0
v = 18	1048572	1.0000	4	68719476736.0

very attractive (v = 6, v = 8)

Note that for the **v = 8 weighted-voting Stiffler ranging sequence**, the **range-clock** component (correlation $\pm$ 981 920 in 1 048 576 chips) is **slightly smaller** **(stronger)** than that of the 1999 JPL ranging sequence (correlation $\pm$ 963 390 **($\pm$ 942 600)** in 1 009 470 chips).

If we use only a single correlator to acquire the 20 bit sequences serially, the normalized total acquisition time is

$$\tau_{acq\text{-}tot} \approx 1.14 + 19 \times 936.1 \approx \mathbf{17\ 787.}$$

which is about 20% smaller than for the 1999 JPL scheme (see slide 27) in which five correlators are used in parallel, one for each component sequence except C1.

The next slide shows the improvement when parallel correlators are used–four correlators seems realistic.

v = 8 weighted-voting Stiffler ranging sequence

# of correlators	normalized total acquisition time
1	17787.0
2	9362.1
3	6553.8
4	4681.6

The **v = 6 weighted-voting Stiffler ranging sequence** has a **range-clock** component (correlation ± 873 392 in 1 048 576 chips) **1.18 dB** **(only 0.33 dB)** **weaker** than that of the 1999 JPL ranging sequence (correlation ± 963 390 **(± 942 600)** in 1 009 470 chips) and gives markedly better acquisition:

# of correlators	normalized total acquisition time
1	3790.1
2	1995.5
3	1397.3
4	998.5

Probability of successful acquisition

Let P_e denote the probability of an incorrect in-phase decision for **bi** against its antipodal alternative, given that the decisions for the previous bit sequences are correct. Because there are 19 bit sequences excluding the range-clock sequence **b0** (which is almost certain to be acquired correctly) and because the ranging sequence is correctly acquired if and only if all 19 decisions are correct, it follows that the probability $\boldsymbol{P_{acq}}$ of **correctly acquiring** the **weighted-voting Stiffler ranging sequence** ranging sequence is well approximated by

$$P_{acq} = (1 - P_e)^{19} \approx 1 - 19P_e$$

which is just the assumption that each of the 19 relevant error events is independent when conditioned on the non-occurrence of the previous error events.

The only bad feature of the **weighted-voting Stiffler ranging-sequence scheme** is its poor spectral properties.

The high-order bit sequences are low frequency sequences:

	Max run 0's	Max run 1's	# of transitions
b1	2	2	524288
b2	4	4	262144
b3	8	8	131072
b4	16	16	65536
b5	32	32	32768
b6	64	64	16384
b7	128	128	8192
b8	256	256	4096
b9	512	512	2048
b10	1024	1024	1024
b11	2048	2048	512
b12	4096	4096	256
b13	8192	8192	128
b14	16384	16384	64
b15	32768	32768	32
b16	65536	65536	16
b17	131072	131072	8
b18	262144	262144	4
b19	524288	524288	2

The “trick” to modifying the **weighted-voting Stiffler ranging-sequence scheme** to obtain good spectral properties is to **decimate** the bit sequences before combining them. This has no effect on the correlation and acquisition properties.

To decimate a length-**L** sequence by **d**, means to take every **d**th digit (cyclically), starting with the first digit, where **d** and **L** must be relatively prime.

Example: **L** = 16, **d** = 11

original sequence	0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1
decimated sequence	0 1 0 0 1 0 0 1 1 0 1 1 0 1 1 0

(Note that 3 $\times$ **d** mod **L** = 1. This is true for all decimations that we will use to create good spectral properties.)

Scrambled weighted-voting Stiffler ranging-sequence scheme

This is the same as the **weighted-voting Stiffler ranging-sequence scheme** except that **all 20 of the bit sequences are decimated by d = 699 051** before the voting to determine the ranging sequence.

N.B. The range-clock sequence **b0** is not changed by decimation!

Proposition: In the decimated version of each of the bit sequences **b1**, **b2**, … , **b19**, the maximum run of 0's has length 2 and the maximum run of 1's has length 2.

The ranging sequence has excellent spectral properties as we will see on slide 51.

All the **scrambled** bit sequences are high frequency sequences, fairly close to the range-clock frequency:

	Max run 0's	Max run 1's	# of transitions
scrambled b1	2	2	524288
scrambled b2	2	2	786432
scrambled b3	2	2	655360
scrambled b4	2	2	720896
scrambled b5	2	2	688128
scrambled b6	2	2	704512
scrambled b7	2	2	696320
scrambled b8	2	2	700416
scrambled b9	2	2	698368
scrambled b10	2	2	699392
scrambled b11	2	2	698880
scrambled b12	2	2	699136
scrambled b13	2	2	699008
scrambled b14	2	2	699072
scrambled b15	2	2	699040
scrambled b16	2	2	699056
scrambled b17	2	2	699048
scrambled b18	2	2	699052
scrambled b19	2	2	699050

Not surprisingly, the ranging sequence itself also has excellent spectral properties.

Scrambled v = 8 weighted-voting Stiffler ranging sequence

The number of zeroes in the sequence is 524 288

The number of ones in the sequence is 524 288

The longest run of zeroes has length 5.

The longest run of ones has length 5.

The number of transitions (0 to 1 and 1 to 0) is 985 308.

Scrambled v = 6 weighted-voting Stiffler ranging sequence

The number of zeroes in the sequence is 524 288

The number of ones in the sequence is 524 288

The longest run of zeroes has length 5.

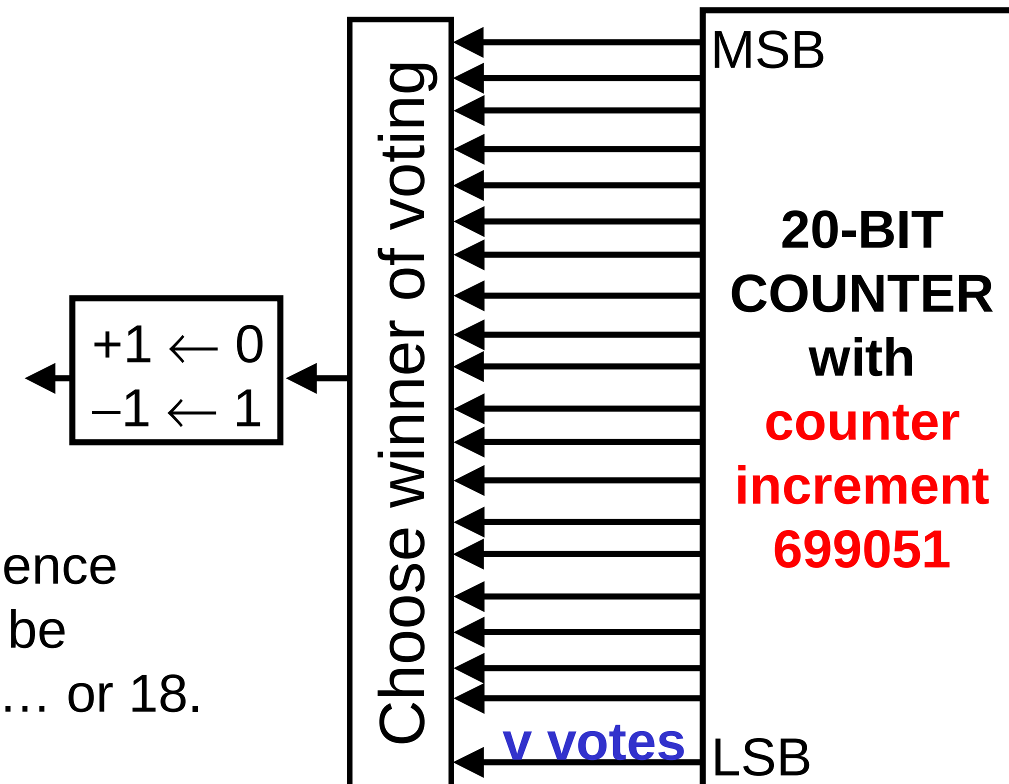
The longest run of ones has length 5.

The number of transitions (0 to 1 and 1 to 0) is 902 792.

The **scrambled weighted-voting Stiffler ranging sequence** is essentially as easy to implement as the previous **weighted-voting Stiffler ranging-sequence**. One needs only to replace the 20-bit counter on slide 37 by a counter that counts in steps of 699 051 (modulo 2^{20}, i.e., ordinary two's-complement logic).

The counter is started in the all-zero state.

±1 ranging sequence



The chips of the LSB sequence have **v votes** where v can be chosen as 2 or 4 or 6 or 8 … or 18.

Acquisition of the **scrambled weighted-voting Stiffler ranging sequence** is essentially the same as for the previous **weighted-voting Stiffler ranging-sequence**.

The only change is that, when several correlators are used in parallel (see slides 40 and 41), then the number of chips that must be skipped in the scrambled scheme is the number that must be skipped in the previous scheme multiplied by 3 modulo 2^{20}. The reason for this is that 3 is the reciprocal of the decimating factor d = 699 051, i.e., $3 \times 699\ 051$ modulo $2^{20} = 1$.

The **scrambled weighted-voting Stiffler ranging-sequence scheme** **generalizes** to ranging sequences of period 2^n for all $n \geq 3$ (with the proviso that, when n is odd, the number of votes **v** assigned to the range-clock sequence must also be odd).

The appropriate decimation factor is

$$d = \frac{2^n + 1}{3} \text{ for odd } n$$

and

$$d = \frac{2^{n+1} + 1}{3} \text{ for even } n.$$

This gives a wide variety of highly efficient and easily implemented ranging schemes.

> The **scrambled weighted-voting Stiffler ranging-sequence scheme** should be a strong candidate for use in future space applications.

Summary Table (Giovanni Boscagli)

PN Sequence	Normalized ACQ Time $\tau_{acq-tot}$	Clock component $10 * Log_{10}(\xi^2)$
JPL 1999	**22081 sec (note 1)**	**-0.4 dB**
Titsworth V4	**10196 sec (note 1)**	**-0.6 dB**
Titsworth V2	**697 sec (note 1)**	**-4.2**
Stiffler V8	**4681.6 sec**	**-0.57**
Stiffler V6	**998.5 sec**	**-1.59**

Note 1: based on Alenia Breadboard architecture (5÷6 correlators)
Note 2: assuming 4 correlators for parallel acquisition.

- The ratio between Titsworth V4 and JPL 1999 normalized acquisition time is 0.46
- The ratio between Titsworth V2 and JPL 1999 normalized acquisition time is 0.031
- **Titsworth V4** and **Stiffler V8** have almost same clock component but we have approximately a factor 2 better for the acquisition time for Stiffler V8.
- **Titsworth V2** and **Stiffler V6** have almost the same acquisition performances (slightly shorter for Titsworth V2) but Stiffler V6 has a much higher (2.6 dB) clock component