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Study on PN ranging codes for future missions

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What we will try to do in this presentation:

- Develop a simple metric, normalized acquisition time, for comparing ranging sequences and their acquisition algorithms.
- Determine the normalized parallel acquisition time for the 1999 JPL ranging-sequence scheme.
- Propose an improvement of the 1999 JPL ranging-sequence scheme.
- Propose an alternative new ranging-sequence scheme.
- Describe effective acquisition procedures for the new ranging-sequence scheme.
- Determine the normalized parallel acquisition time for the new ranging-sequence scheme.
- Specify the spectrum of the new ranging-sequence signal.

In all practical ranging systems, the **±1 periodic ranging sequence** is acquired by the receiver as the result of **correlations** between the received sequence and **certain ±1 periodic sequences** that are related in some manner to the ranging sequence and that we will call **probing sequences**.

The probing sequences may be subsequences of the ranging sequence or they may be related to the ranging sequence in less direct ways, e.g., the ranging sequence might be the result of some kind of vote among the chips of all the probing sequences at the same position.

For each probing sequence, one or more correlations are made to determine when this probing sequence is "in-phase" with the received sequence over the segment of the received sequence where the correlation is performed.

EXAMPLE:

The first periods of the six “**component sequences**” for the 1999 JPL scheme are:

C1 = +1 -1

C2 = +1 +1 +1 -1 -1 +1 -1

C3 = +1 +1 +1 -1 -1 -1 +1 -1 +1 +1 -1

C4 = +1 +1 +1 +1 -1 -1 -1 +1 -1 -1 +1 +1 -1 +1 -1

C5 = +1 +1 +1 +1 -1 +1 -1 +1 -1 -1 -1 -1 +1 +1 -1 +1 +1 -1 -1

C6 = +1 +1 +1 +1 +1 -1 +1 -1 +1 +1 -1 -1 +1 +1 -1 -1 +1 -1 +1 -1 -1 -1 -1

The ranging sequence is obtained by combining the chips of the six periodic component sequences at the same position in the manner that the result is +1 if C1 has a +1 at that position or if all five of the sequences C2, C3, C4, C5 and C6 have a +1 at that position.
The period is $2 \times 7 \times 11 \times 15 \times 19 \times 23 = 1009470$.

J. B. Berner, J. M. Layland, P. W. Kinman and J. R. Smith, "Regenerative Pseudo-Noise Ranging for Deep-Space Applications", Telecommunications and Mission Operations (TMO) Progress Report 42-137, Jet Propulsion Laboratory, Pasadena, CA, May 15, 1999.

The **probing sequences** are the six component sequences and their distinct cyclic shifts. The number of probing sequences is thus
2 + 7 + 11 + 15 + 19 + 23 = 77.

The correlations of the six +1/-1 probing sequences C1, C2, C3, C4, C5 and C6 with one period of the +1/-1 ranging sequence are as follows:

C1: 963390 -963390
C2: 46080 0 0 0 0 0 0
C3: 46080 0 0 0 0 0 0 0 0 0 0
C4: 46080 0 0 0 0 0 0 0 0 0 0 0 0 0 0
C5: 46080 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
C6: 46080 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

The probing sequence that is the second right cyclic shift of C3 gives the following correlations: 0 0 46080 0 0 0 0 0 0 0 0
and so on.

Because the period of the ranging sequence is 1009470, we see from the correlations with C1 that the ranging sequence is closely approximated by C1 itself, which is an alternating +1/-1 sequence.

ACQUISITION OF A PROBING SEQUENCE

The “in-phase” position of probing sequence is acquired by first **correlating** the received noisy ranging sequence (starting at some unknown point), over a specified number of periods of the ranging sequence, **with all the cyclic shifts of that probing sequence**, then making a decision as to which cyclic shift is in-phase with the signal component of the received sequence.

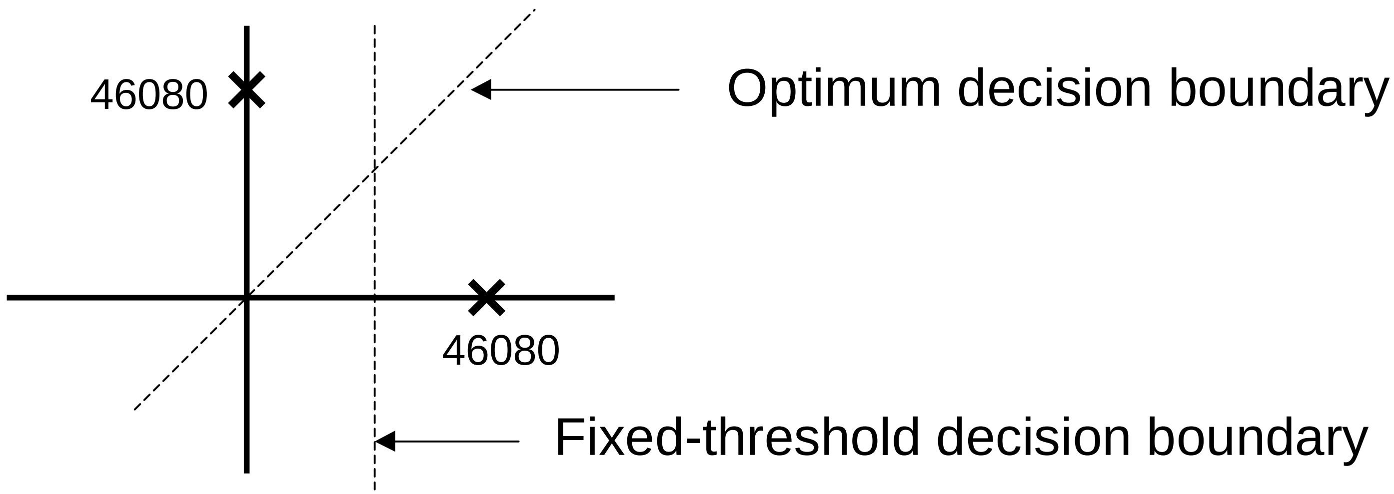
Exception: For the alternating +1/-1 sequence, one correlation suffices because the correlation with this sequence is just the negative of the correlation with its cyclic shift.

Under the assumption of additive white Gaussian noise, the **optimum decision rule** is to **choose the in-phase position** of the probing sequence as **corresponding to the cyclic shift that gave the largest correlation**.

J. M. Wozencraft and I. M. Jacobs, *Principles of Communication Engineering*. New York: Wiley, 1965.

Warning: Comparison to a fixed threshold when making the decision on the in-phase position for **orthogonal alternatives** incurs a **3 dB penalty**.

Example (using the 1999 JPL ranging sequence):
Consider the acquisition of the probing sequences that are the cyclic shifts of C2. The correlations over one period for +1 +1 +1 -1 -1 +1 -1 and its cyclic shift -1 +1 +1 +1 -1 -1 +1 in the absence of noise are as follows:



The signal points are farther from the optimum decision boundary by a factor of $\sqrt{2}$ compared to the fixed-threshold boundary.

Probability of error in an “in-phase” decision between two **antipodal** sequences of length K chips with energy E_C per chip when the noise is additive white Gaussian noise with two-sided power spectral density $N_0/2$:

$$P_e = Q\left(\sqrt{2KE_C / N_0}\right)$$

(Wozencraft & Jacobs, p. 250)

where

$$Q(\alpha) = \frac{1}{\sqrt{2\pi}} \int_{\alpha}^{\infty} e^{-x^2/2} dx$$

(Wozencraft & Jacobs, p. 49)

Equivalently, the **number K of chips needed** for a given P_e with **antipodal** signals is

$$K = \frac{\left[Q^{-1}(P_e)\right]^2}{2E_C / N_0}.$$

This is the basis for our evaluation of the efficiency of probing sequences.

The **number *K* of chips needed** for a given P_e with **antipodal** signals is rather **insensitive to** the chosen value of P_e for any specified value of the signal-to-noise ratio.

P_e	K (for E_C/N_0 of –36 dB) (or $2E_C/N_0$ of –33 dB)
10^{-9}	71,707.
10^{-10}	80,643.
10^{-11}	89,595.
10^{-12}	98,570.
10^{-13}	107,563.
10^{-14}	116,583.

Assuming that we want P_e to be about 10^{-12}, we see that we can assume that ***K* is about 100,000 chips**. This is the figure we will use in our comparisons.

We now specify the **three parameters of an arbitrary probing sequence** that will allow us to determine how many chips of the **received sequence** must be correlated with this probing sequence to obtain the specified P_e.

The key is that KE_C in the expression for P_e is **proportional to the squared Euclidean distance** between the antipodal signals of length K chips. Thus, we need only determine what fraction of this squared Euclidean distance (or equivalently what fraction of the **signal-to-noise ratio**) is achieved between the probing sequence and the out-of-phase cyclic shift to which it is compared.

The three parameters are:

ξ = **average percentage correlation**
γ = **correlation scale factor**
ν = **interleaving parameter**

Average percentage correlation:

C_{in} = in-phase correlation

n_{act} = number of chips actually correlated with the probing sequence to obtain C_{in}

$$\xi = \frac{C_{in}}{n_{act}}$$

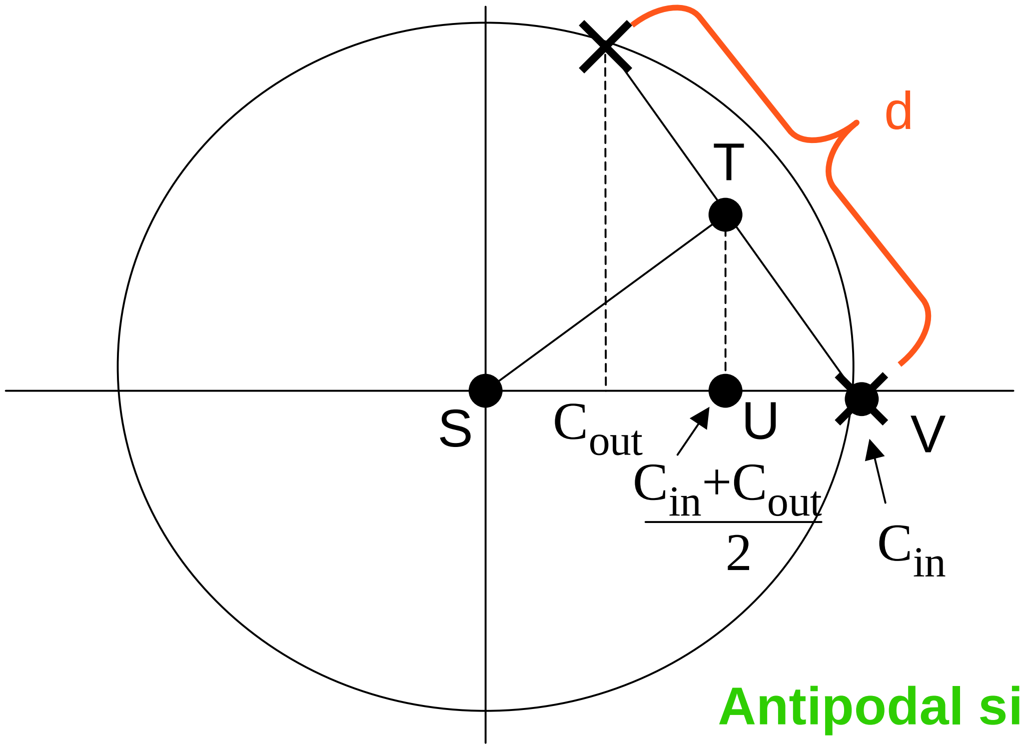
Example: Recall for the 1999 JPL probing sequence C3, the correlations were C3: 46080 0 0 0 0 0 0 0 0 0 0. This correlation was done over all n_{act} = 1009470 chips in one period.

$$\Rightarrow \quad \xi = 46080/1009470 = 0.04565$$

Because the signal amplitude at the receiver is proportional to C_{in} while the noise power is proportional to n_{act}, the **squared Euclidean distance** between signal points for an in-phase decision is ξ^2 times that for antipodal sequences of the same length n_{act}.

Example: (continued) $\xi^2 = 0.00208$ (loss of 26.8 dB)

Correlation scale factor



STV and TUV are similar triangles.

$$\Rightarrow \quad \frac{\tfrac{1}{2}\, d}{C_{in} - \frac{C_{in}+C_{out}}{2}} = \frac{C_{in}}{\tfrac{1}{2}\, d}$$

so that

$$\boxed{d^2 = 2\, C_{in}(C_{in} - C_{out})}$$

Antipodal signals ($C_{out} = - C_{in}$) give $d^2 = 4\, C_{in}^2$

Defining the **correlation scale factor** λ as the ratio of the **squared Euclidean distance** between the **actual signals** and between **antipodal signals of the same energy**, we obtain

$$\boxed{\lambda = \frac{C_{in} - C_{out}}{2\, C_{in}}}$$

For **orthogonal signals** ($C_{out} = 0$), $\lambda = ½$ (3 dB loss)

Example: (continued) Recalling again that for the 1999 JPL probing sequence C3, the correlations were

C3: 46080 0 0 0 0 0 0 0 0 0 0.

Each in-phase decision is thus a decision between **orthogonal signals** (C_{out} = 0) so that λ = ½ (3 dB loss).

Interleaving parameter

Suppose that the chips of the probing sequence are correlated only with every ν^{th} chip of the ranging sequence. Then the equivalent number of chips that orthogonal signals must be correlated to get the **same squared Euclidean distance** is only **$1/\nu$** times the number of chips that must be used for the actual signals.

Example: (continued) For the 1999 JPL probing sequence C3 (and indeed for all the probing sequences in this ranging scheme) there is no interleaving, i.e.,
$\nu = 1$ so that also $1/\nu = 1$ (0 dB loss)

An hypothetical example: Suppose $\nu = 5$
Then $1/\nu = 0.2$ (7 dB loss)

Normalized correlation time

We define the **normalized correlation time, τ_{cor},** of a probing sequence with parameters ξ, λ, and ν to be

$$\boxed{\tau_{cor} = \frac{\nu}{\lambda\xi^2}}$$

To find the correlation time in chips of the received sequence, we just need to multiply τ_{cor} by the parameter K, the number of such chips needed for a given P_e with antipodal signals

Example: (continued) For the 1999 JPL probing sequence C3 (and indeed for all the probing sequences in this ranging scheme) except C1, the normalized correlation time is

$$\tau_{cor} = \frac{1}{½ \times 0.00208} \approx 960$$

For E_C/N_0 of –36 dB and $P_e \approx 10^{-12}$, $K \approx$ 100,000 chips $\Rightarrow$ the correlation of C3 requires about 96,000,000 received chips.

Normalized acquisition time

To **acquire** a particular probing sequence, one needs in principle to correlate the received sequence with each cyclic shift of that probing sequence, then choose the cyclic shift with greatest correlation. Thus, it is natural to define the **normalized acquisition time** of the probing sequence as

$$\tau_{acq} = n_s \, \tau_{corr}$$

where n_s is the number of cyclic shifts of the probing sequence that must be correlated with the received sequence.

Remark: n_s is the number of distinct cyclic shifts of the probing sequence, **except** when some cyclic shift of the probing sequence is the antipodal sequence, in which case n_s is half the number of distinct cyclic shifts as the antipodal correlations need not be done.

Example: (continued) For the 1999 JPL probing sequence C3, there are seven distinct cyclic shifts so $\tau_{acq} = 7 \times \tau_{corr} \approx 7 \times 960 = 6{,}720$.

Example: For the 1999 JPL probing sequence C1, there are two distinct cyclic shifts but these are antipodal sequences so that $n_s = 1$ correlations suffice. Thus $\tau_{acq} = \tau_{corr}$.
The correlations are C1: 963390 -963390 when done over all $n_{act} = 1009470$ chips in one period. Thus $\xi = 963390/1009470 = 0.95435$ and $\xi^2 = 0.91079$. $\lambda = 1$ (antipodal signals) and $\nu = 1$ (no interleaving) so that

$$\tau_{cor} = \frac{1}{0.91079} \approx 1.10$$

which is smaller by a factor of about 875 than the correlation time for the probing sequence C3. Moreover, the acquisition time for C1 is $\tau_{acq} = \tau_{corr} \approx 1.10$, which is smaller by a factor of about 6,120 than the acquisition time for C3.

Normalized parallel acquisition time

We now consider the time required to acquire the phase of the **entire ranging sequence**, i.e., to acquire all of the probing sequences, when the availability of multiple correlators allows some of the correlations to be carrier out in parallel. This parallel acquisition time will depend on the particular acquisition algorithm used.

We use the 1999 JPL ranging scheme as an **example to illustrate how parallel acquisition time is calculated**. In this scheme, it is forseen that

1) the probing sequence C1 (also called the "clock component") will first be acquired by itself.

2) thereafter, the probing sequences C2, C3, C4, C5 and C6 will be acquired in parallel.

1) Acquisition of C1 requires a normalized time of $\tau_{acq} \approx 1.10$.

2) The normalized time for parallel acquisition of the probing sequences C2, C3, C4, C5 and C6 is equal to the largest normalized acquisition time of these five probing sequences, which is that for C6, namely $\tau_{acq} = 23 \times \tau_{corr} \approx 23 \times 960 = 22{,}080$.

Thus the **normalized parallel acquisition time** for the entire ranging sequence is

$$\tau_{par\text{-}acq} \approx 1.10 + 22{,}080 \approx 22{,}081.$$

For E_C/N_0 of –36 dB and $P_e \approx 10^{-12}$, $K \approx 100{,}000$ chips $\Rightarrow$ the parallel acquisition of the entire ranging sequence requires about 2,208,100,000 received chips.

The 1999 JPL ranging scheme can be improved by changing the **rule for forming the ranging sequence** from the six component sequences C1, C2, C3, C4, C5 and C6 to the following:

> Combine the chips of the six periodic component sequences at the same position by **weighted voting** with C1 given 4 votes and the other sequences 1 vote each.

The correlations of C1, C2, C3, C4, C5 and C6 with one period (1,009,470 chips) of this +1/-1 ranging sequence are as follows:

C1: 942600 -942600

C2: 66870 -6930 -6930 -6930 -6930 -6930 -6930

C3: 66870 -4158 -4158 -4158 -4158 -4158 -4158 -4158 -4158 -4158 -4158

C4: 66870 -2970 -2970 -2970 -2970 -2970 -2970 -2970
-2970 -2970 -2970 -2970 -2970 -2970 -2970

C5: 66870 -2310 -2310 -2310 -2310 -2310 -2310 -2310 -2310 -2310 -2310
-2310 -2310 -2310 -2310 -2310 -2310 -2310 -2310

C6: 66870 -1890 -1890 -1890 -1890 -1890 -1890 -1890 -1890 -1890 -1890
-1890 -1890 -1890 -1890 -1890 -1890 -1890 -1890 -1890 -1890
-1890 -1890

The parameters of these six probing sequences (under the same assumptions as before) are

	ξ	ξ^2	λ	ν	τ_{corr}	τ_{acq}
C1:	0.9334	0.8719	1	1	1.1	1.1
C2:	0.0662	0.0044	0.5518	1	413.0	2,891.
C3:	0.0662	0.0044	0.5311	1	429.1	4.720.
C4:	0.0662	0.0044	0.5222	1	436.4	6,546.
C5:	0.0662	0.0044	0.5173	1	440.6	8,371.
C6:	0.0662	0.0044	0.5141	1	443.3	10,195.

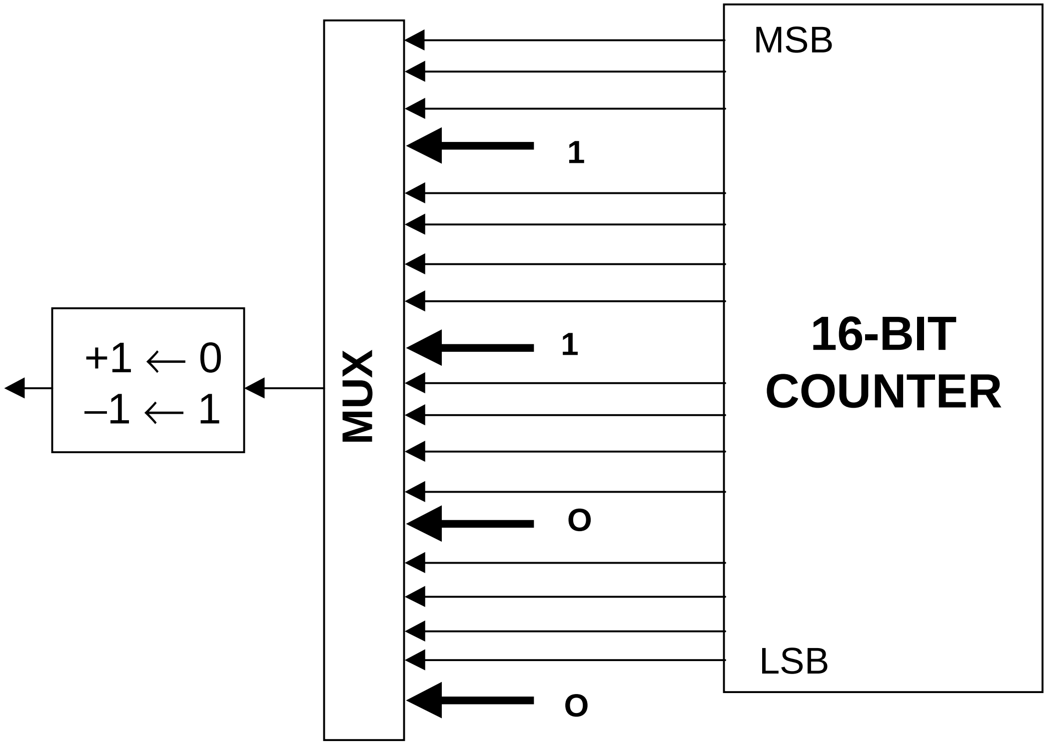
(Note that the clock component is virtually as strong as it was in the original 1999 JPL ranging sequence.)

The **normalized parallel acquisition time** for the entire ranging sequence is

$$\tau_{par\text{-}acq} \approx 1.1 + 10{,}195 \approx 10{,}196$$

which is less than half that of the original 1999 JPL ranging sequence.

A proposed alternative ranging scheme:



Generation of the Ganz-Hiltgen-Massey (GHM) ranging sequence.

The period of the GHM ranging sequence is $20 \times 2^{16} = 1{,}310{,}720$.

J. Ganz, A. P. Hiltgen, and J. L. Massey, Final Report ESTEC Contract No. 8579/89/NL/DG Definition of Codes for User Ranging via DRSS at Ka-Band, Institute for Signal- and Information Processing, ETH Zurich, March 1990.

Stucture of interleaved frame as binary digits:

0 b0 b1 b2 b3 0 b4 b5 b6 b7 1 b8 b9 b10 b11 1 b12 b13 b14 b15

Stucture of **interleaved frame** as +1/-1 chips:

+1 c0 c1 c2 c3 +1 c4 c5 c6 c7 -1 c8 c9 c10 c11 -1 c12 c13 c14 c15

The probing sequence **+1 +1 -1 -1** interleaved to depth ν = 5, or equivalently the sequence

+1 0 0 0 0 +1 0 0 0 0 -1 0 0 0 0 -1 0 0 0 0

and its 19 cyclic shifts is used to **acquire the boundaries** of the interleaved frame, i.e, to acquire frame synchronization.

Each of these probing sequences is antipodal to its cyclic shift by 10 positions so that **only 10 correlations** need to be carried out to acquire frame synchronization.

The correlations of this probing sequence with the entire GHM ranging sequence over one period are as follows:

262144 0 0 0 0 0 0 0 0 0 -262144 0 0 0 0 0 0 0 0 0

Note that this probing sequence is orthogonal to eighteen of its cyclic shifts and antipodal to one of its cyclic shifts.

The parameters of the probing sequence **+1 +1 -1 -1** interleaved to depth $\nu = 5$ are thus

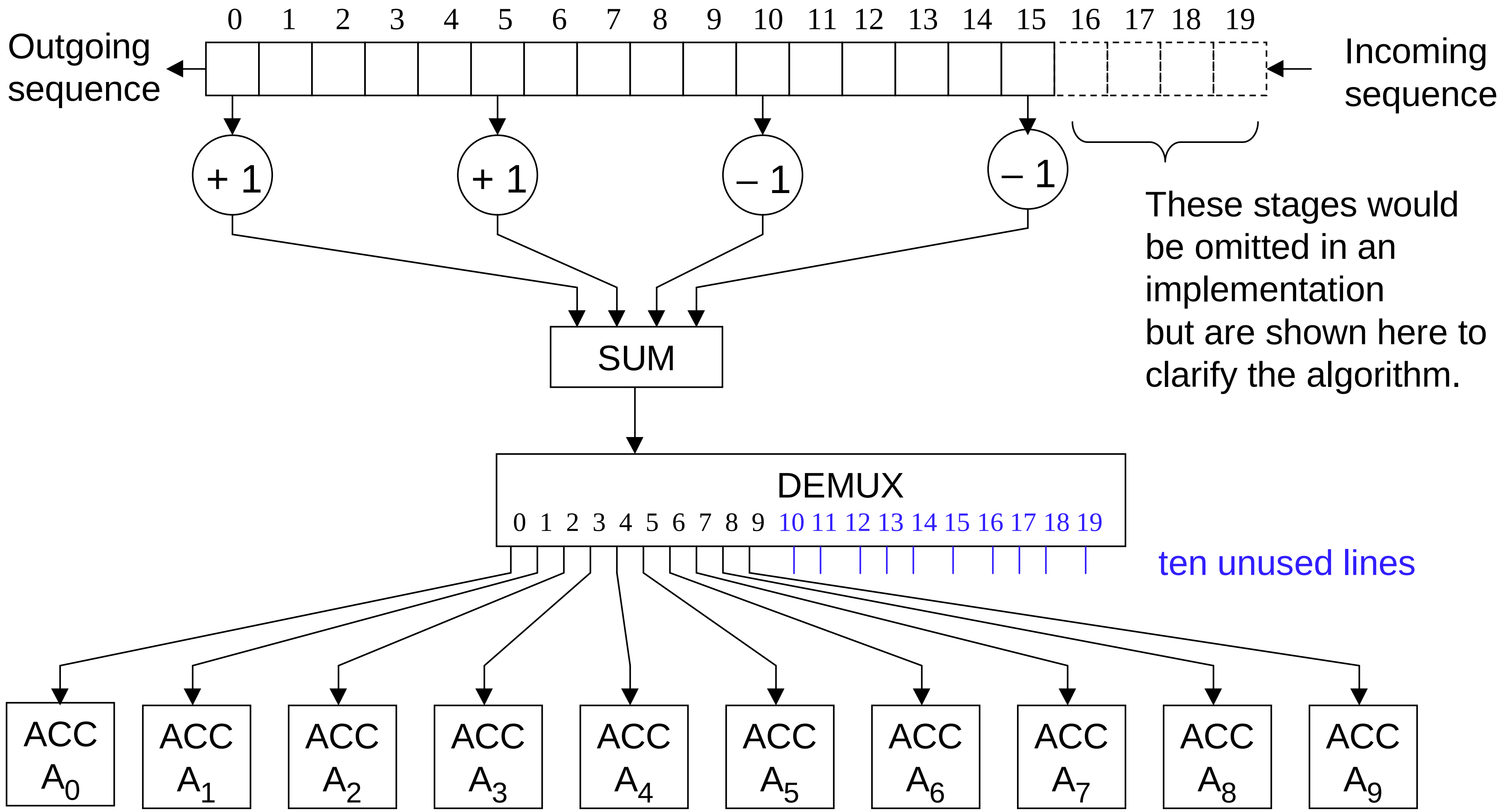
ξ	ξ^2	λ	ν	τ_{corr}
1	1	½	5	10

We will show that it is easy to carry out in parallel all ten correlations needed to acquire the probing sequence **+1 +1 -1 -1** interleaved to depth $\nu = 5$ so that the **normalized time for frame acquisition** is

$$\tau_{acq} = \tau_{corr} = 10.$$

We now show three different ways to perform this frame acquisition.

Method 1 of frame acquisition: Simultaneous correlation with one period of the probing sequence +1 +1 -1 -1 interleaved to depth ν = 5.



Decision procedure for Method 1 of frame acquisition:

1) After summing over ML chips (where L is the period of the ranging sequence and M is the number of periods to be used for frame acquisition), the accumulated value with ***maximum magnitude***, say A_μ, is selected.

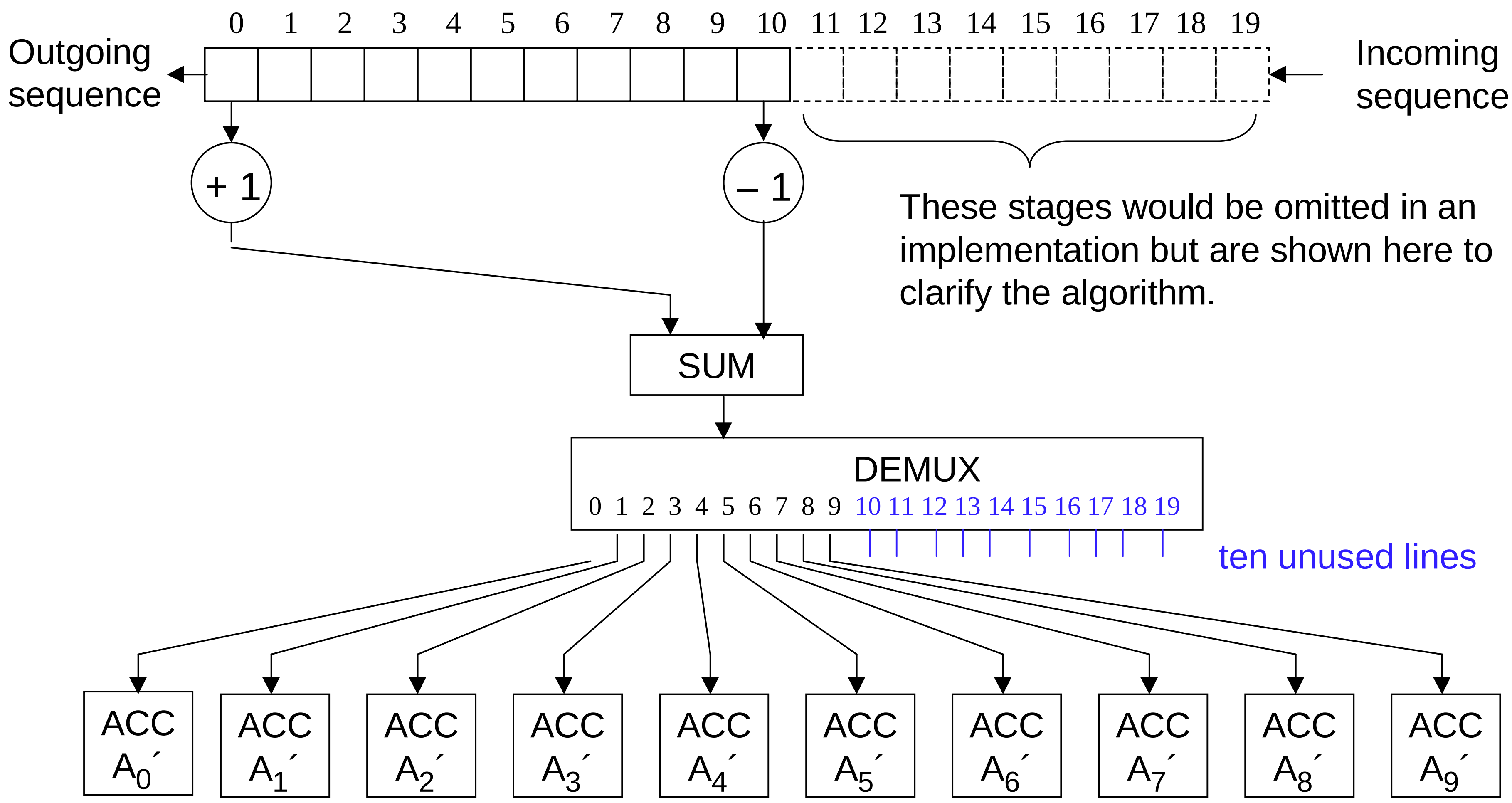
2) The **decision** is that the chip currently in position D of the shift register is a chip in position 0 of the interleaved frame where

$$D = \mu \text{ if } A_\mu \geq 0$$

and

$$D = \mu + 10 \text{ if } A_\mu < 0.$$

Method 2 of frame acquisition: Simultaneous correlation with the auxiliary sequence +1 -1 interleaved to depth ν = 5, followed by conversion to correlation values for Method 1.



Decision procedure for Method 2 of frame acquisition:

1) After summing over ML chips (where L is the period of the ranging sequence and M is the number of periods to be used for frame acquisition), the accumulated values are converted as follows

$$A_i = A_{i+5}' + A_i'$$

and

$$A_{i+5} = A_{i+5}' - A_i'$$

for $i = 0, 1, \ldots, 4$.

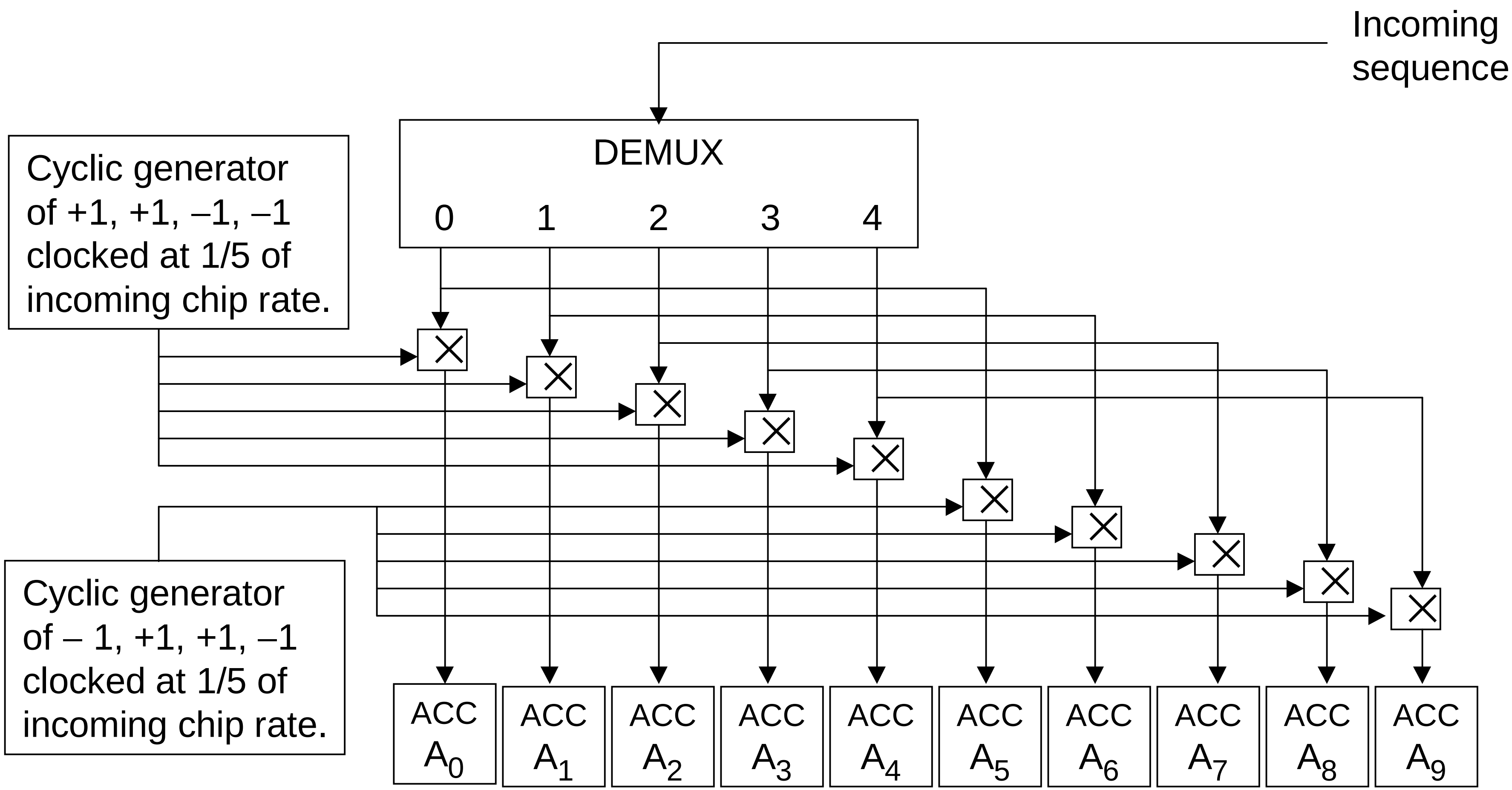
2) The **decision** is that the chip currently in position D of the shift register is a chip in position 0 of the interleaved frame where

$$D = \mu \text{ if } A_\mu \geq 0$$

and

$$D = \mu + 10 \text{ if } A_\mu < 0.$$

Method 3 of frame acquisition: Sequential correlation with the probing sequence +1 +1 -1 -1 interleaved to depth ν = 5.



Decision procedure for Method 3 of frame acquisition:

(Note that each five chips of the incoming sequence are treated as a multiplexed set and converted to one chip in each of **five "slow clock" sequences**. These five chips result in one addend to each of the ten accumulators.)

1) After summing over ML chips (where L is the period of the ranging sequence and M is the number of periods to be used for frame acquisition), the accumulated value with **maximum *magnitude***, say A_μ, is selected.

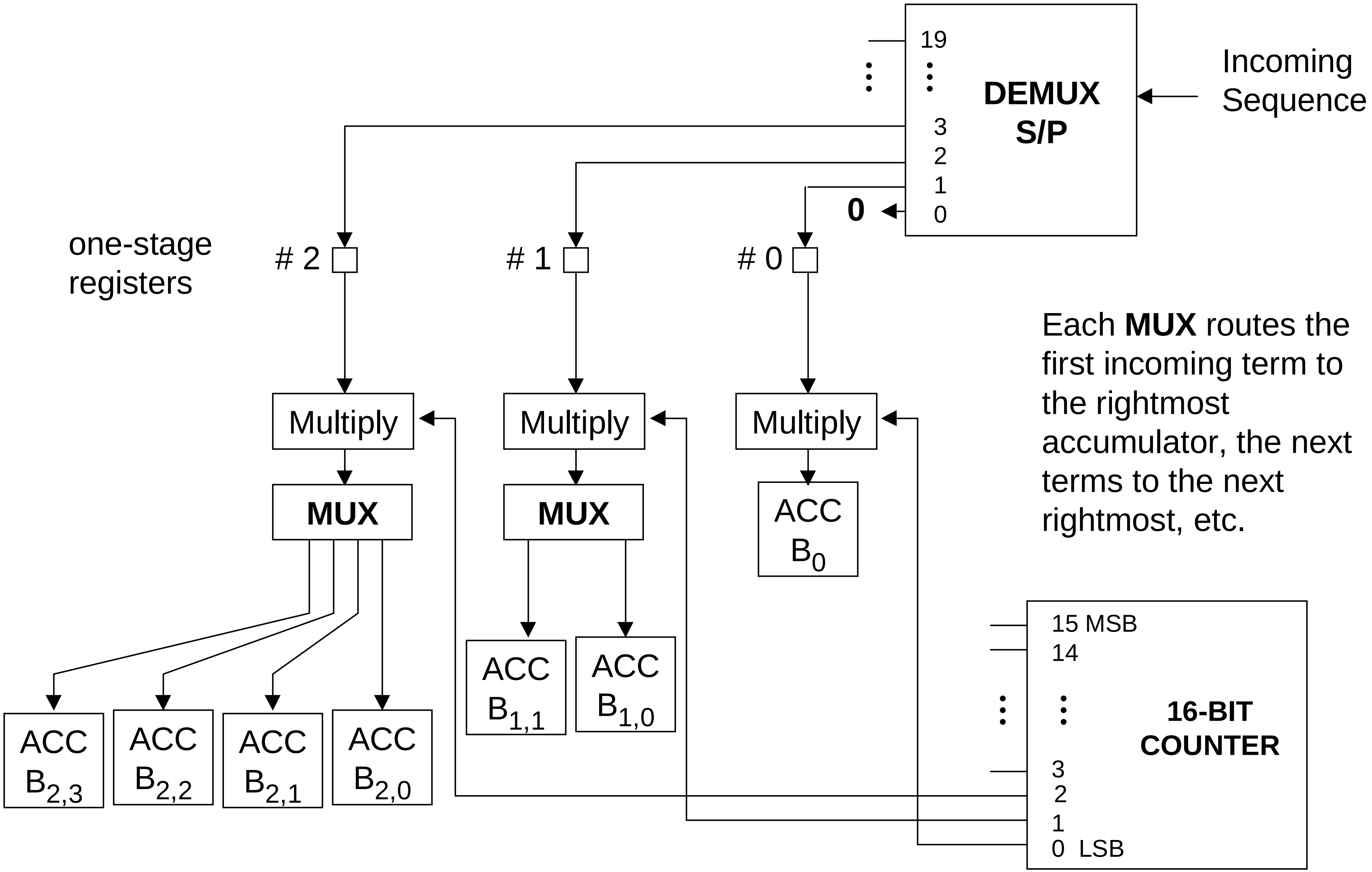
2) The **decision** is that the first chip of the first full interleaved frame is D chips beyond the jumping-in point where correlation began where

$$D = \mu \text{ if } A_\mu \geq 0$$

and

$$D = \mu + 10 \text{ if } A_\mu < 0.$$

Parallel acquisition of first three counter-bit sequences:



Each **MUX** routes the first incoming term to the rightmost accumulator, the next terms to the next rightmost, etc.

Before beginning counter-bit-sequences acquisition, incoming chips have been skipped (if necessary) so that bit #0 of the counter-bit frame is routed to output line 0 of the DEMUX.

Decision rules for acquisition of first three counter-bit sequences:

After summing over ML chips

- if $B_0 < 0$, set $b_0 = 1$. Otherwise, set $b_0 = 0$.

- if $b_0 = 0$, set $B_1 = + B_{1,0} + B_{1,1}$.
- if $b_0 = 1$, set $B_1 = - B_{1,0} + B_{1,1}$.
- if $B_1 < 0$, set $b_1 = 1$. Otherwise, set $b_1 = 0$.

- if $b_1\ b_0 = 0\ 0$, set $B_2 = + B_{2,0} + B_{2,1} + B_{2,2} + B_{2,3}$.
- if $b_1\ b_0 = 0\ 1$, set $B_2 = - B_{2,0} + B_{2,1} + B_{2,2} + B_{2,3}$.
- if $b_1\ b_0 = 1\ 0$, set $B_2 = - B_{2,0} - B_{2,1} + B_{2,2} + B_{2,3}$.
- if $b_1\ b_0 = 1\ 1$, set $B_2 = - B_{2,0} - B_{2,1} - B_{2,2} + B_{2,3}$.
- if $B_2 < 0$, set $b_2 = 1$. Otherwise, set $b_2 = 0$.

Prepare for parallel processing of next three counter sequences:

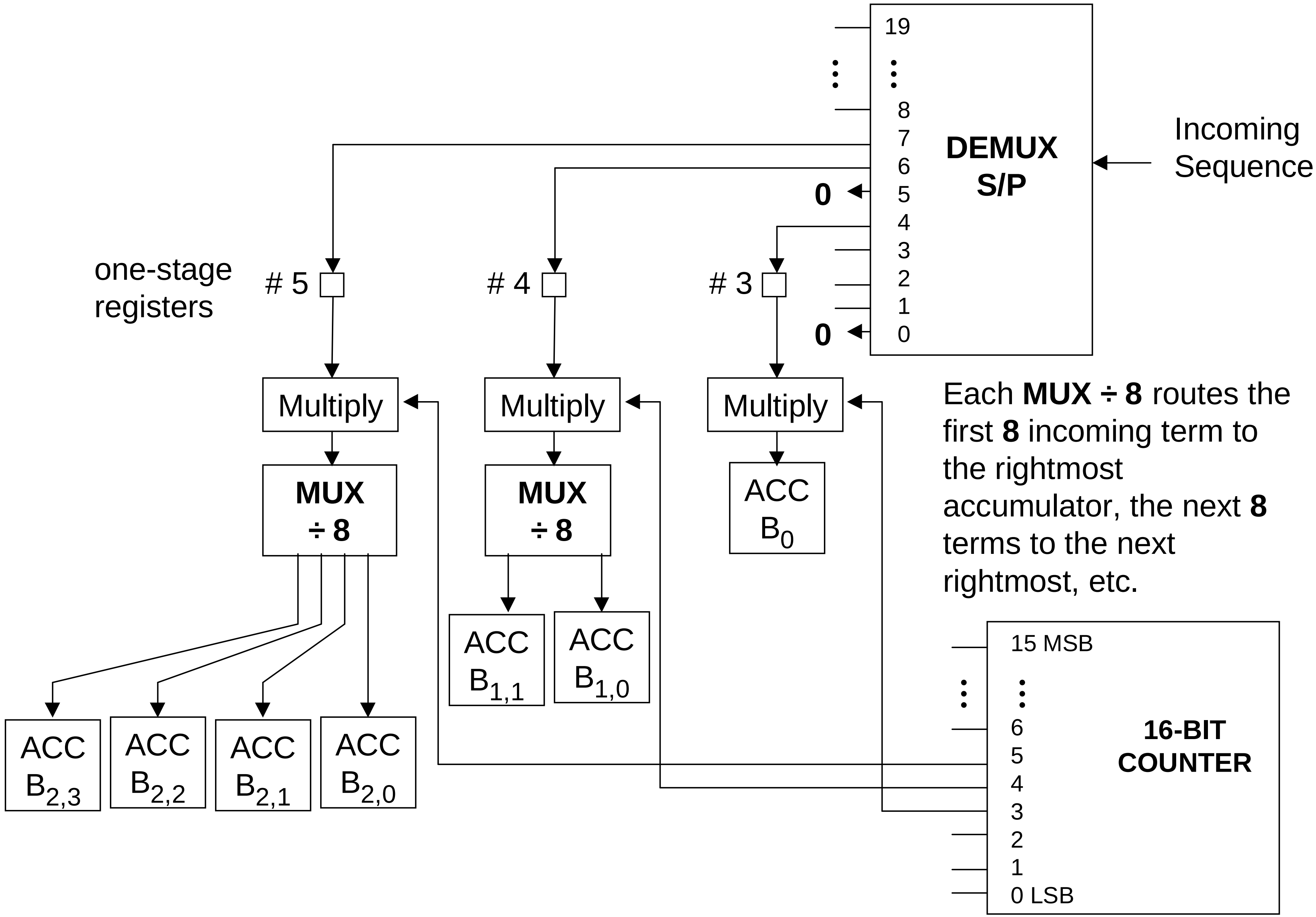
Clock the DEMUX and one-stage registers to **accept**

$$\mathbf{b = 4b_2 + 2b_1 + b_0}$$

further incoming frames before processing the next three counter-bit sequences.

(The counter-bit chips currently in the one-stage registers # 2, # 1 and # 0 will now all be +1 (binary 0) if no decision error has occurred.)

Parallel acquisition of second three counter sequences:



Each **MUX ÷ 8** routes the first **8** incoming term to the rightmost accumulator, the next **8** terms to the next rightmost, etc.

Decision rules for acquisition of second three counter-bit sequences:

- if $B_0 < 0$, set $b_0 = 1$. Otherwise, set $b_0 = 0$.

- if $b_0 = 0$, set $B_1 = + B_{1,0} + B_{1,1}$.
- if $b_0 = 1$, set $B_1 = - B_{1,0} + B_{1,1}$.
- if $B_1 < 0$, set $b_1 = 1$. Otherwise, set $b_1 = 0$.

if $b_1\ b_0 = 0\ 0$, set $B_2 = + B_{2,0} + B_{2,1} + B_{2,2} + B_{2,3}$.

- if $b_1\ b_0 = 0\ 1$, set $B_2 = - B_{2,0} + B_{2,1} + B_{2,2} + B_{2,3}$.
- if $b_1\ b_0 = 1\ 0$, set $B_2 = - B_{2,0} - B_{2,1} + B_{2,2} + B_{2,3}$.
- if $b_1\ b_0 = 1\ 1$, set $B_2 = - B_{2,0} - B_{2,1} - B_{2,2} + B_{2,3}$.
- if $B_2 < 0$, set $b_2 = 1$. Otherwise, set $b_2 = 0$.

Prepare for parallel processing of next three counter sequences:

Clock the DEMUX and one-stage registers to **accept $2^3(4b_2+2b_1+b_0)$ $= 8(4b_2+2b_1+b_0)$ further incoming frames** .

The parameters of the sixteen counter-bit probing sequences are:

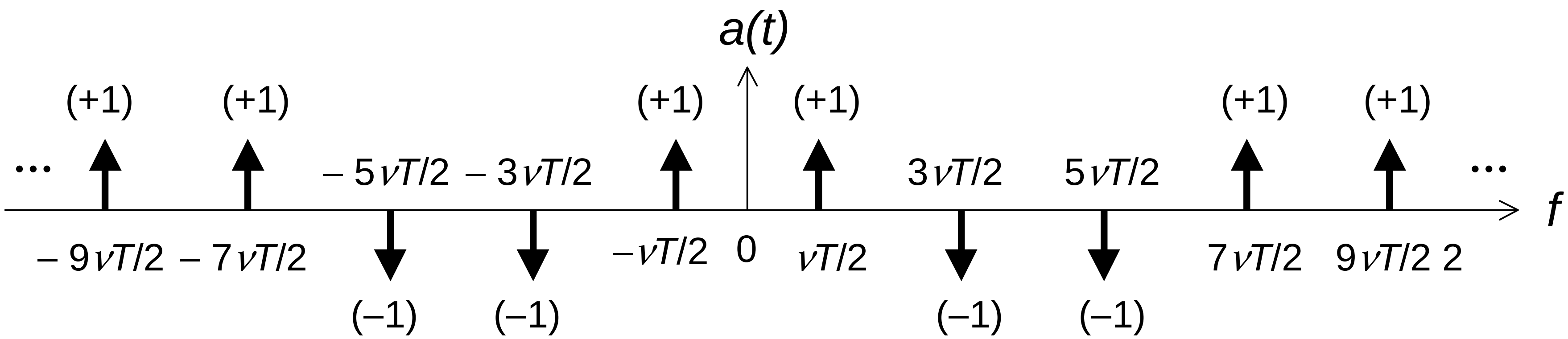
ξ	ξ^2	λ	ν	τ_{corr}
1	1	1	20	20

The most natural choice for parallel acquisition is to acquire four counter-bit sequences in parallel, and do this four times.
The normalized time for parallel acquisition of each group of four counter-bit sequences is $\tau_{acq} = \tau_{corr} = 20$.
The **normalized parallel acquisition time** for the entire GHM ranging sequence is thus

$$\tau_{par\text{-}acq} \approx 10 + 4 \times 20 = 90.$$

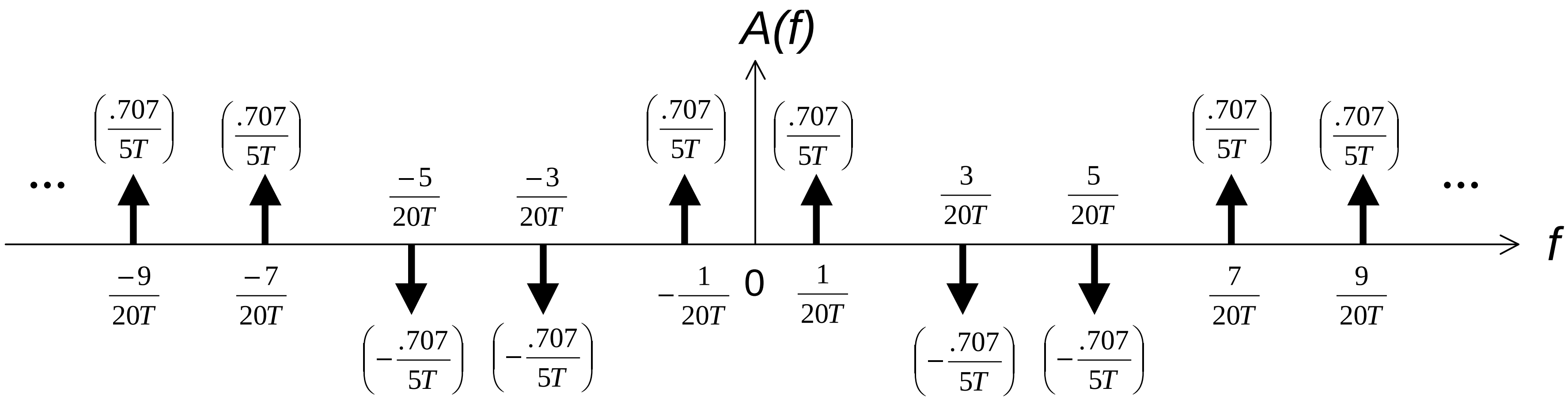
Spectra of the GHM ranging sequence

The fixed sequence in each interleaving frame that is used for the acquisition of the frame boundaries, considered as an impulse train, is

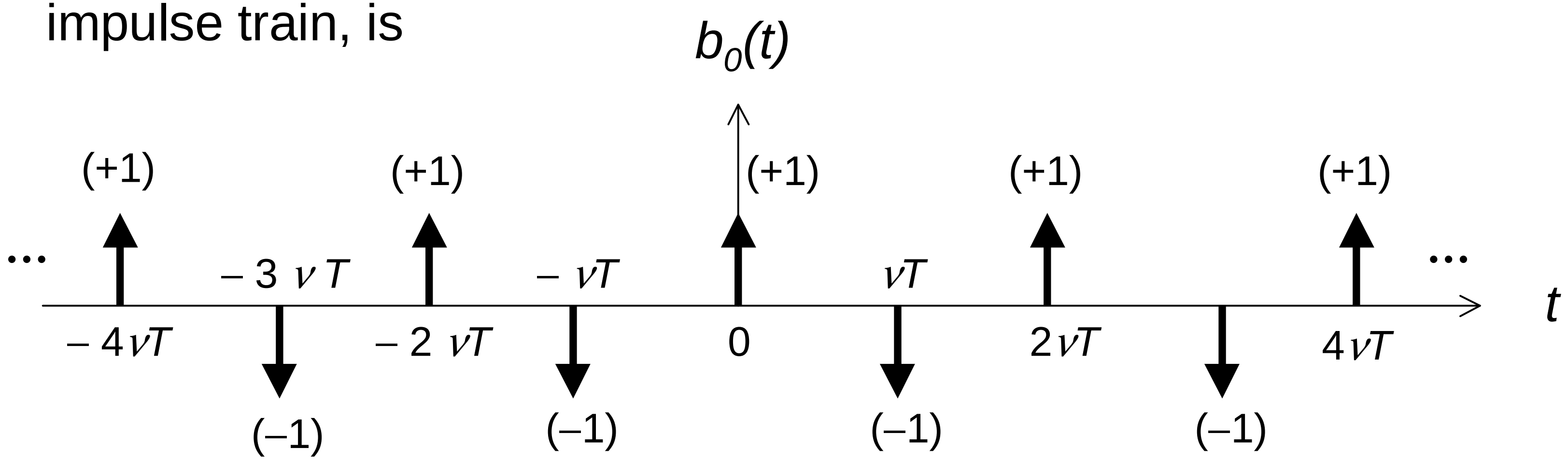


where $\nu = 5$ and T is the chip time.

The Fourier transform of this impulse train *a(t)* is

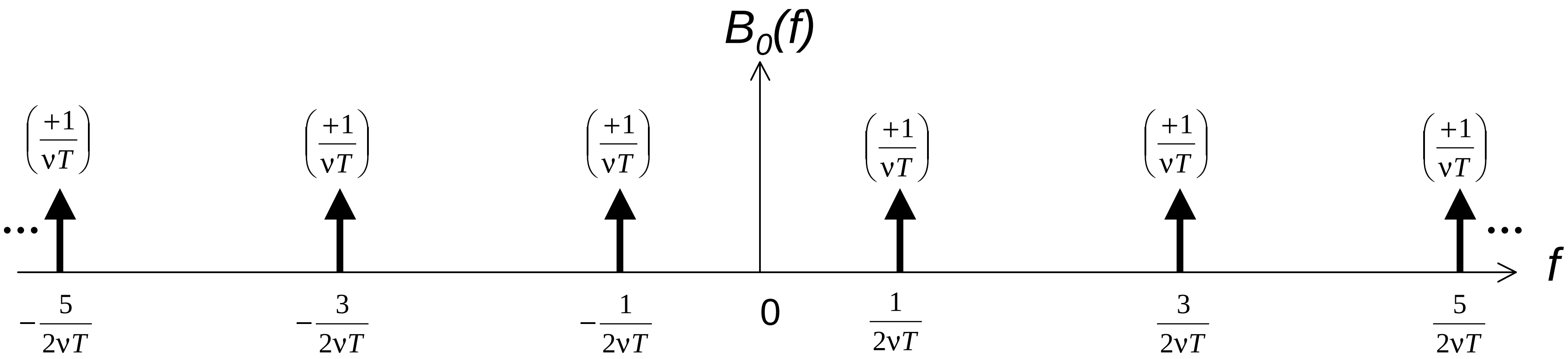


The least-significant-counter-bit sequence, considered as an impulse train, is

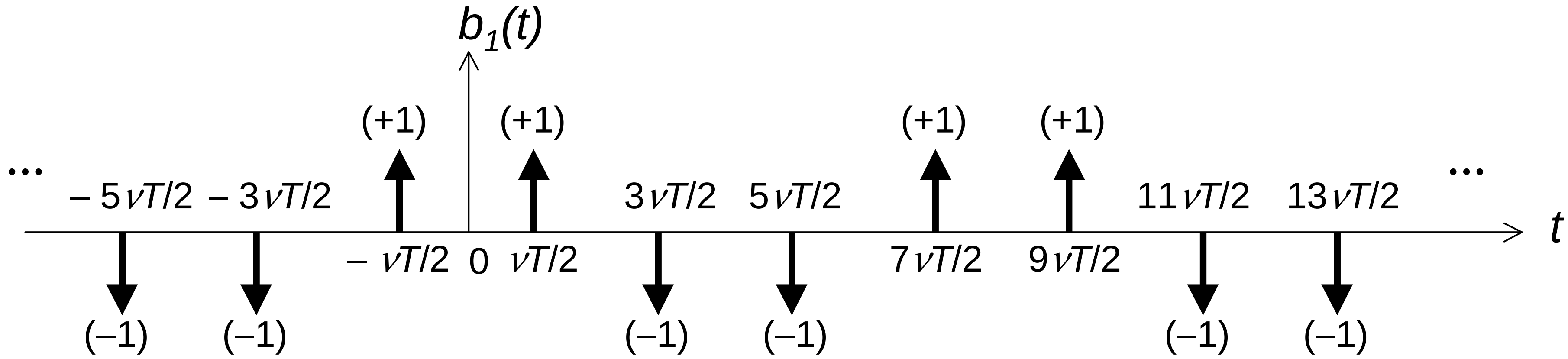


where $\nu = 20$ and T is the chip time.

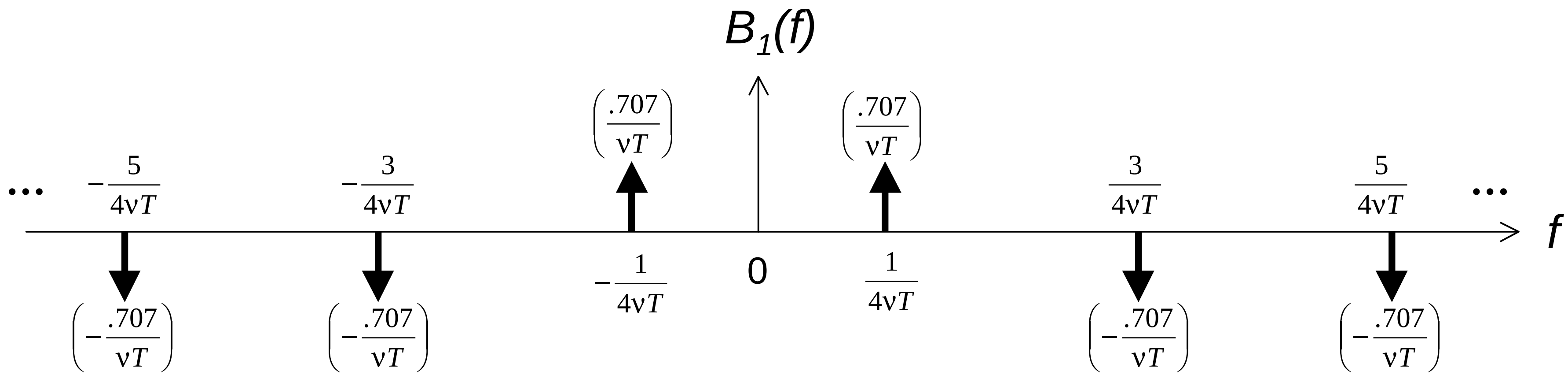
The Fourier transform of this impulse train $b_0(t)$ is



The second-least-significant-counter-bit sequence, considered as an impulse train, is



where $\nu = 20$ and T is the chip time.
The Fourier transform of this impulse train $b_1(t)$ is



The counter-bit sequence with runs of 2^k identical bits, considered as an impulse train, $b_{k+1}(t)$, with the origin taken at the midpoint of a run of 2^k impulses with area +1, has the Fourier transform

$$B_{k+1}(f) = \frac{1}{\nu T} \sum_{i=-\infty}^{+\infty} c_i \, \delta\left(f - \frac{2i-1}{2^k \nu T}\right)$$

where $\nu = 20$ and T is the chip time. The coefficients in this expression are given by

$$c_i = \prod_{l=1}^{k-1} \cos\left(\frac{2i-1}{2^l}\frac{\pi}{2}\right)$$

for $k > 1$.

The coefficients c_i of the line spectrum $B_k(f)$ have even symmetry, i.e., $c_i = c_{-i}$ and have period 1 for $k = 0$ and period 2^{k+1} for $k > 0$.

The following are the first 8 coefficients of each of the line spectra:

k	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8
0	1.0000							
1	0.7071	-0.7071						
2	0.6533	-0.2706	0.2706	-0.6533				
3	0.6407	-0.2250	0.1503	-0.1274	0.1274	-0.1503	0.2250	-0.6407
4	0.6376	-0.2153	0.1326	-0.0985	0.0809	-0.0709	0.0653	-0.0628
5	0.6369	-0.2130	0.1286	-0.0928	0.0731	-0.0608	0.0525	-0.0465
6	0.6367	-0.2124	0.1276	-0.0914	0.0713	-0.0586	0.0498	-0.0434
7	0.6366	-0.2123	0.1274	-0.0911	0.0709	-0.0581	0.0492	-0.0427
8	0.6366	-0.2122	0.1273	-0.0910	0.0708	-0.0579	0.0490	-0.0425
9	0.6366	-0.2122	0.1273	-0.0910	0.0707	-0.0579	0.0490	-0.0425

k	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8
10	0.6366	-0.2122	0.1273	-0.0909	0.0707	-0.0579	0.0490	-0.0424
11	0.6366	-0.2122	0.1273	-0.0909	0.0707	-0.0579	0.0490	-0.0424
12	0.6366	-0.2122	0.1273	-0.0909	0.0707	-0.0579	0.0490	-0.0424
13	0.6366	-0.2122	0.1273	-0.0909	0.0707	-0.0579	0.0490	-0.0424
14	0.6366	-0.2122	0.1273	-0.0909	0.0707	-0.0579	0.0490	-0.0424
15	0.6366	-0.2122	0.1273	-0.0909	0.0707	-0.0579	0.0490	-0.0424
16	0.6366	-0.2122	0.1273	-0.0909	0.0707	-0.0579	0.0490	-0.0424
17	0.6366	-0.2122	0.1273	-0.0909	0.0707	-0.0579	0.0490	-0.0424
18	0.6366	-0.2122	0.1273	-0.0909	0.0707	-0.0579	0.0490	-0.0424
19	0.6366	-0.2122	0.1273	-0.0909	0.0707	-0.0579	0.0490	-0.0424

Work in progress and/or planned future work:

- Design an effective method for obtaining chip synchronization with the GHM ranging sequence and carefully work out the performance of this method.

- Quantify the effects of filtering on the performance of the chip synchronization procedure and on the performance of the sequence acquisition procedure.