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EUROPEAN SPACE AGENCY
Marco Lanucara

Reliable carrier phase synchronization with GMSK+PN - configuration aspects

Abstract

This paper is an answer to following action AI_22-01 originated at the Spring 2022 CCSDS SLS-RFM Working Group Meeting Minutes: "*Provide material regarding GMSK receiver tracking performance with Doppler rate, and selection of the appropriate loop parameters for inclusion in the 413.1-G-2 Green Book (based on inputs SLS-RFM_22-02, SLS-RFM_22- 06, SLS-RFM_21, SLS-RFM_21-21, and SLS-RFM_21-22)*".

References

RD-1 CCSDS 401.0-B-32, Radio Frequency and Modulation Systems—Part 1: Earth Stations and Spacecraft.

RD-2 Vassallo, E., and M. Visintin. "Carrier phase synchronization for GMSK signals." International journal of satellite communications 20.6 (2002): 391-415.

RD-3 Gardner, Floyd M. Phaselock techniques. *John Wiley & Sons*, 2005.

RD-4 810-005, Rev. E, DSMS Telecommunications Link Design Handbook, 34-m and 70-m Telemetry Reception, 207, Rev. A,

NOTE: As this material is intended to potentially constitute an annex to "CCSDS 413.1-G-2, Simultaneous Transmission of GMSK Telemetry and PN Ranging", such reference is not explicitly repeated in the text when referring to it. So, for example, reference to Eq. (19) in the above Green Book is simply referenced as Eq. (19). Furthermore, new equation and section numbers defined below are temporary, to be adapted when embedding the annex in the Green Book. Finally, symbols already defined in the Green Book (e.g. T_s) are not redefined here.

Annex: Reliable carrier phase synchronization with GMSK+PN - configuration aspects

1. Introduction

The recovery of the (suppressed) carrier phase from a Gaussian Minimum Shift Keying (GMSK) modulated signal requires the implementation of a carrier phase synchronizer, whose system parameters must be set to cope with available signal to noise ratio (SNR), Doppler dynamics, phase noise, simultaneous transmission of a pseudo noise (PN) ranging signal. A key performance is related to the distribution of the phase error committed by the carrier synchronizer during tracking. In particular, the jitter (defined as square root of the variance) and mean value of the phase error are of interest, due to their impact on telemetry performance. Beyond the induced degradation, a large phase error, either in jitter or in mean or in both, will lead to the occurrence of phase slips, or complete loss of lock. Therefore, containing the amount of phase error, both in jitter and mean, is a critical objective for robust communications, requiring the application of established models which relate the system parameters to the phase error distribution.

The above section 3.4.6 provides a procedural approach for determining the carrier phase jitter in the case of simultaneous transmission of GMSK and PN ranging implemented according to 2.4.22A or 2.4.22B of [RD-1], depending on the selected product BT_s to be 0.25 or 0.5 respectively. The last step of the approach, i.e. the use of Eq. (19) in section A3.2, is applicable to a specific carrier synchronizer implementation, as clarified in the next section. The present annex complements the content of section 3.4.6 and A3.2 in the following aspects:

1. It generalizes the determination of carrier phase jitter to different types of carrier synchronizer;
2. it provides an approach for determining the mean phase error, on top of the jitter;
3. it provides guidelines for evaluating the robustness of the receiver configuration from the point of view of minimizing the probability of phase slips or unlock.

Within this analysis the phase noise induced by local oscillators, or turned around during two-way coherent operations, or due to other impairments (e.g. interaction with solar plasma), is neglected assuming that its impact is negligible vs. other effects, however it may be included with established methods from [RD-3] and [RD-4] whenever the above assumption cannot be made.

2. Evaluation of the carrier phase error jitter for a generic carrier synchronizer

The procedure described in section 3.4.6 starts from the total power-to-noise spectral density P_{T}/N_0 and based on known configuration parameters, determines, by a procedure which is not repeated here, a Carrier Phase Jitter Loss. Such loss (defined as positive in dB) reduces the value of the parameter $P_{\mathrm{T}}T_{\mathrm{s}}/N_0$ to be used in Eq. (19) of section A3.2 to compute the phase jitter[1]. The Eq. (19) is applicable to the maximum a posteriori (MAP) estimation of the carrier phase, implemented by a carrier synchronizer as in Figure 2 in [RD-2] with low SNR ratio approximation for the hyperbolic tangent. The approach can be generalized to different carrier synchronizers by re-writing Eq. (19) as follows, based on Eq. (17) of [RD-2]

$$\sigma^2 = \sigma_{\varsigma}^2 (2\pi)^2 = \frac{B_{\mathrm{L}}T_{\mathrm{s}}}{S_{\mathrm{L}} \cdot P_{\mathrm{T}}\, T_{\mathrm{s}}/N_0} \tag{1}$$

where S_{L} is customarily called "Squaring Loss", defined as negative in dB (a terminology and sign convention which are preserved in the present annex, to align with existing literature). The value of the Squaring Loss depends upon the specific modulation, the ratio[2] $P_{\mathrm{T}}\, T_s/N_0$ as well as the selected carrier synchronizer. In particular the Eq. (1) is equivalent to Eq. (19) when setting

$$S_L = \frac{1}{\delta^2} \frac{P_T\, T_s/N_0}{1 + 2(P_T\, T_s/N_0)} \tag{2}$$

with δ defined by the adoption of either $BT_{\mathrm{s}} = 0.25$ or $BT_{\mathrm{s}} = 0.5$ as per Table A-5. The above expression of the Squaring Loss is valid for the previously mentioned MAP carrier synchronizer with low SNR ratio approximation. However, Eq. (1) can be applied to any synchronizer architecture, once the function S_{L} is known in closed form or by simulations. In presence of simultaneous GMSK and PN ranging signals the value of $P_{\mathrm{T}}\, T_s/N_0$ to be used for the computation of the Squaring Loss and in Eq. (1) must be reduced by the Carrier Phase Jitter Loss.

When extending the approach described in section 3.4.6 to a generic carrier synchronizer, additional information is required to model and predict the tracking loop performance, i.e. the potential dependency of the loop parameters which appear in Eq. (1) from signal-to-noise ratio

[1] Indeed Eq. (19) is written for the case of GMSK-only modulation, with the implicit prescription to reduce the $P_{\mathrm{T}}\, T_{\mathrm{s}}/N_0$ by the Carrier Phase Jitter Loss in the case of GMSK+PN, as clarified in the detailed computations in section 3.4.6.1.

[2] In absence of PN ranging modulation, and neglecting waveform distortions or other non-idealities, the term $P_{\mathrm{T}}\, T_s/N_0$ is essentially equal to the ratio E_s/N_0 between energy of a coded symbol and noise spectral density, and some quoted references use the latter denomination in contexts of GMSK-only.

conditions. For example, in the case of the MAP carrier synchronizer assumed by Eq. (19), the loop parameters are independent from the ratio $P_{\mathrm{T}}\,T_{\mathrm{s}}/N_0$. However, a dependency from the same ratio, possibly reduced in presence of simultaneous PN ranging, must be considered in the study case in section 4.

3. Evaluation of the carrier phase error mean for a generic carrier synchronizer

The parameter representative of Doppler dynamics relevant to this annex is the rate of variation of the received signal carrier frequency which is induced by the relative acceleration between the transmitting and receiving antennas, defined as $\dot{f}$ in Hz/s. It is assumed that such acceleration is constant, and that the carrier synchronizer exhibits a tracking behavior equivalent to a second-order type 2 phase locked loop (PLL) with natural frequency ω_{n} and damping factor ζ. The noise bandwidth B_{L} for such loop is obtained from the above parameters ([RD-3])

$$B_{\mathrm{L}} = \frac{\omega_{\mathrm{n}}}{2}\left(\zeta + \frac{1}{4\zeta}\right) \tag{3}$$

The mean of the phase error, defined as θ_a, depends, for the considered second order type 2 tracking loop, on the input frequency rate $\dot{f}$ and on the natural frequency of the tracking loop ω_{n} according to the following relation ([RD-3], for a phase detector with sinusoidal characteristic)

$$\sin\theta_{\mathrm{a}} = \frac{2\pi\dot{f}}{\omega_{\mathrm{n}}^2} \tag{4}$$

A normalized frequency rate parameter $\gamma = 2\pi\dot{f}/\omega_{\mathrm{n}}^2$ is also presented in the test results in section 5, whose value corresponds to θ_{a} for small values of the error. Furthermore θ_{a} and γ are always shown as unsigned quantities irrespectively of the sign of $\dot{f}$ in the test results in section 5. When applying Eq. (4), a potential dependency of the loop parameters, and in particular ω_{n}, from $P_{\mathrm{T}}\,T_{\mathrm{s}}/N_0$ must be considered.

4. Study case: decision-directed carrier synchronizer

As an example of the approach reported in the previous sections, the carrier tracking performance of a decision-directed GMSK carrier synchronizer is analyzed in this section. The synchronizer has an architecture consistent with the one in Figure 9 in [RD-2]. The first step is retrieving the performance models for the selected modulation and carrier synchronizer. Within this sample study case either $BT_{\mathrm{s}} = 0.25$ or $BT_{\mathrm{s}} = 0.5$ are used, with chip rate R_{RG} close (however not identical) to the coded symbol rate R_{s}, peak modulation index $m_{\mathrm{RG}} = 0.444$ radians and sinusoidal pulse. Either a T2B or a T4B sequence is used for the PN ranging sequence.

The key models required for the communications link design are the dependency of the Squaring Loss and of the loop parameters from $P_{\mathrm{T}}\,T_{\mathrm{s}}/N_0$. For the considered decision-directed scheme, and for GMSK only, the Squaring Loss assumes the values reported in Table 1 ([RD-2])

Table 1: Squaring Loss S_L as a function of $P_T T_s/N_0$ for a GMSK decision-directed synchronizer (from [RD-2])

$P_T T_s/N_0$ [dB]	S_L [dB]	
	GMSK $BT_s = 0.5$ dec. dir.	GMSK $BT_s = 0.25$ dec. dir.
-10	-12.47	-16.86
-8	-10.03	-13.42
-6	-8.26	-11.20
-4	-6.32	-9.30
-2	-4.48	-6.92
0	-2.91	-5.27
2	-1.82	-3.49
4	-0.85	-2.02
6	-0.15	-0.95
8	-0.08	-0.18
10	-0.07	-0.09

Concerning the dependency of the loop parameters from $P_T T_s/N_0$, Figure 1 reports the reduction factor to be applied to the configured values valid for large SNR, which are defined as B_{L0}, ω_{n0} and ζ_0, for the specific synchronizer implementation, modulations of interest and particularized to the case that $\zeta_0 = 1$. It is important to emphasize that different synchronizer architectures and implementations lead to different curves, including the case of no dependency.

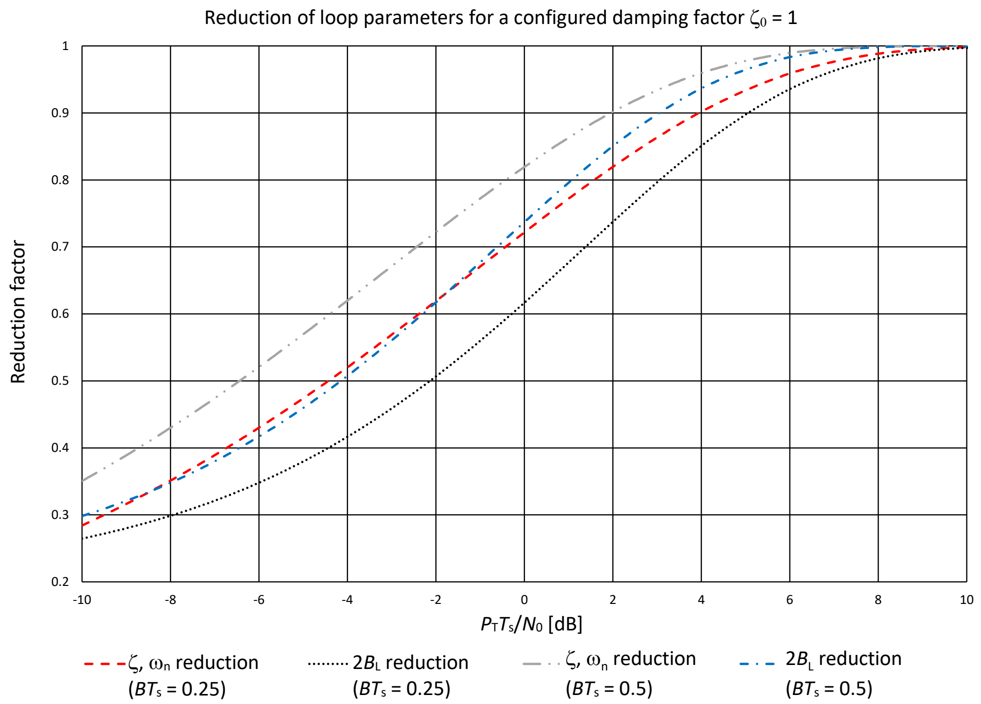

Figure 1: Reduction of the loop parameters for the study-case synchroniser implementation, for GMSK only and for a configured damping factor equal to one.

The above models are applied, together with the methodology described in section 3.4.6, to two sample scenarios representative of missions in Near Earth or Deep Space. The assumptions for the two missions are reported in Table 2. The Overall Telemetry Loss and Carrier Phase Jitter Loss are computed based on the procedure outlined in section 3.4.6, with steps that are not repeated here. The above procedure leads to the reduced E_s/N_0 and reduced $P_T T_s/N_0$, the latter to be used for analysing the performance of the carrier synchroniser. The Squaring Loss is computed from interpolation of the relevant columns in Table 1, and the actual value of the loop parameters is retrieved from the curve represented in Figure 1. The phase jitter is subsequently computed according to Eq. (1). When assuming a certain frequency rate, the mean phase error is computed according to Eq. (4).

Table 2: Link budget analysis for carrier synchronisation performance in the case of a decision-directed synchroniser

Mission sample scenario	Near Earth	Deep Space
P_T/N_0 [dBHz]	64.5	55.5
GMSK modulation	BT_s = 0.25	BT_s = 0.5
Information bit rate [Mbit/s]	1.250	0.167
Coding	turbo (k=8920, r=1/4)	turbo (k=8920, r=1/6)
Coded symbol rate [Msymbol/s]	5	1
PN ranging chip rate [Mchip/s]	5.033	1.003
PN ranging modulation index [rad]	0.444	0.444
PN ranging code	T4B	T2B
E_s/N_0 without PN ranging [dB]	-2.51	-4.51
Overall Telemetry Loss with PN ranging [dB]	0.62	0.48
Reduced E_s/N_0 with PN ranging [dB]	-3.1	-5.0
Carrier Phase Jitter Loss with PN ranging [dB]	0.38	0.45
Reduced $P_T T_s/N_0$ with PN ranging [dB]	-2.9	-5.0
Squaring Loss S_L [dB]	-7.9	-7.3
Configured B_{L0} [Hz]	500	525
Configured ζ_0	1	1
B_L reduction factor	0.46	0.46
ζ, ω_n reduction factor	0.57	0.57
Noise bandwidth B_L [Hz]	230	241
$B_L T_s$	4.6E-05	2.4E-04
Damping factor ζ	0.57	0.57
Natural frequency ω_n [1/s]	455	479
Expected phase error jitter [rad]	0.023	0.064
Expected mean phase error in presence of 0.5 kHz/s freq. rate [rad]	0.015	0.014
Expected mean phase error in presence of 2 kHz/s freq. rate [rad]	0.061	0.055

5. Tests with an operational system

This section presents laboratory tests conducted with an operational system, to validate the methodology illustrated in section 3.4.6 and complemented by this annex. The operational system employs a decision-directed carrier synchronization algorithm, for which the Squaring Loss model reported in Table 1 approximately applies. Furthermore, the synchronizer exhibits the dependency of the loop parameters from $P_{\mathrm{T}} T_{\mathrm{s}}/N_0$ which is shown in Figure 1. The results are summarized in Table 3 and Table 4 which report the conditions applicable to each test case. In all tests GMSK+PN modulation is used, with chip rate $R_{\mathrm{RG}} \approx R_{\mathrm{s}}$, ranging modulation index $m_{\mathrm{RG}} = 0.444$ and sinusoidal pulse. The tests in Table 3 are conducted with $BT_{\mathrm{s}} = 0.25$ and T4B PN sequence, whereas $BT_{\mathrm{s}} = 0.25$ and $BT_{\mathrm{s}} = 0.5$ as well as both T2B, T4B sequences are used in the tests of Table 4. The tests in Table 3 aim at studying the occurrence of phase slips during tracking, whereas the tests in Table 4 demonstrate successful tracking in a high Doppler rate scenario, with a frequency rate of around 10.5 kHz/s and with configurations aiming at maximizing the mean phase error (or equivalently the factor γ).

In particular, concerning the tests in Table 4, with $BT_{\mathrm{s}} = 0.25$ correct carrier phase tracking as well as telemetry and ranging operations are established with a maximum mean phase error between 0.2 and 0.336 radians (depending on the coded symbol rate) at a reduced $P_{\mathrm{T}} T_{\mathrm{s}}/N_0$ of around -3 dB. In almost noise free condition ($P_{\mathrm{T}} T_{\mathrm{s}}/N_0$ larger than 9 dB) a maximum mean phase error of 0.377 radians is achieved. With $BT_{\mathrm{s}} = 0.5$ correct carrier phase tracking as well as telemetry and ranging operations are established with a maximum mean phase error of 0.293 radians at a coded symbol rate of 1 Msymbol/s and at a reduced $P_{\mathrm{T}} T_{\mathrm{s}}/N_0$ of around -5 dB. In almost noise free condition ($P_{\mathrm{T}} T_{\mathrm{s}}/N_0$ larger than 9 dB) a maximum mean phase error of 0.476 radians is achieved. The configurations exhibiting a large mean phase error are not necessarily recommended for actual operations, in view of the degradation induced onto the telemetry performance and because of their proximity to configurations triggering phase slips or unlock. Such configurations are only meant to provide experimental upper bounds for supportable mean phase errors during tracking for the operational system under test, which may be useful when analyzing mission phases with extremely high accelerations. Finally all tests in Table 3 and Table 4 show an excellent agreement between predicted and measured carrier phase jitter and mean phase error, thus confirming the reliability of the employed models of Table 1 and Figure 1.

A comparison between $BT_s = 0.25$ and $BT_s = 0.5$ in terms of carrier tracking performance shows that $BT_s = 0.5$ is consistently better than $BT_s = 0.25$ by around 2 dB, meaning that, for the same coded symbol rate and configured loop parameters B_{L0}, ω_{n0} and ζ_0, the carrier jitter and mean phase error assume very similar values for the two modulations when $P_T T_s/N_0$ with $BT_s = 0.25$ is larger by approximately the above distance in dB with respect to $BT_s = 0.5$ (for example one can compare test cases A2 and H2 or A4 and H4 in Table 4). Such experimental fact is consistent with the behavior of the Squaring Loss of Table 1 and of the loop parameters reduction factor of Figure 1 as functions of $P_T T_s/N_0$. It is important to emphasize that such behavior is not only a feature of the decision-directed synchronizer analyzed here and in section 4, but is also applicable to the previously mentioned MAP carrier synchronizer with low SNR ratio approximation for which Eq. (19) applies, as it can be established by inspection of the same Eq. (19), or by looking to [RD-2] results. Such inferior performance in carrier tracking is to be traded with the better spectral occupancy of $BT_s = 0.25$ with respect to $BT_s = 0.5$, for example leading to an advantage of 6% to 8% in occupied bandwidth for the cases studied in this and the previous section.

6. Configuration guidelines

This last section of the annex provides high level guidelines for defining a robust configuration of a GMSK receiver from the point of view of carrier phase tracking. The guidelines can be summarized in the following procedure:

1. Retrieve the performance models for the selected modulation and carrier synchronizer, as a minimum the dependency of Squaring Loss and (potentially) loop parameters from $P_T T_s/N_0$.
2. Define the operating point in terms of coded symbol rate R_s, ratio P_T/N_0, GMSK+PN configuration, adopted channel coding and frequency rate $\dot{f}$.
3. By means of the link budget analysis illustrated in section 3.4.6, compute the reduced E_s/N_0 and $P_T T_s/N_0$ while PN ranging is active. It is assumed that in such configuration adequate margin is obtained for the telemetry and ranging functions at the desired error rate and delay jitter performance respectively.
4. Define the maximum tolerable phase error jitter and mean, based on accepted telemetry performance degradation and to ensure robust acquisition and tracking. This point is deepened at the end of the section.
5. Define a baseline configuration of the loop parameters.
6. Compute the reduction factor of the loop parameters at the reduced $P_T T_s/N_0$ (if any, depending upon the synchronizer implementation), and determine the actual B_L, ω_n and ζ.
7. Compute the squaring loss S_L at the reduced $P_T T_s/N_0$.
8. Compute σ^2 based on Eq. (1), and θ_a based on Eq. (4).
9. Verify that maximum phase error jitter and mean are not exceeded, otherwise go back to above step 5 to attempt a new configuration.
10. If the approach is successful, test the found configuration.

The important step 4 in the above procedure is related to the definition of tolerable phase error jitter and mean. The reason to contain jitter and mean is twofold: 1) ensuring robust acquisition and tracking of the carrier phase, and 2) containing the degradation onto the subsequent telemetry and ranging functions. Within this discussion only the first aspect is addrssed, whereas the second must be tackled for every individual set of mission requirements with methods available in literature (see for example the methodology described in [RD-4], however not covering directly GMSK).

The setting of upper bounds for phase error jitter and mean must be based on a combination of published results and experimental characterization with the selected operational system, as a minimum commensurate with the one described in sections 4 and 5. The Figure 2 illustrate such analysis based on the data from Table 3 and Table 4 (only for the range of the reduced $P_{\mathrm{T}}T_{\mathrm{S}}/N_0$ between -5 and -3 dB, which is of practical interest). The circles originate from configurations leading to continuous lock and absence of phase slip, concurrently to the successful provision of telemetry and ranging functions. Conversely the crosses originate from configurations where phase slips occurred, or lock could not be sustained. The square data point in Figure 2 is taken from Eq. (25) in [RD-4], which proposes an upper bound on the carrier phase jitter to avoid half-cycle phase slips in the case of BPSK. As expected, such data point is fitting very well at the edge of the successful tracking region which emerges from the tests with GMSK.

The first remark is that it is not possible to identify a unique upper bound for e.g. the mean phase error. For example, for a carrier phase jitter approaching 0.14 radians, only a very small, or negligible, mean phase error will be tolerated. Conversely, if the carrier phase jitter is small, e.g. well below 0.1 rad, some amount of mean phase error will be tolerated from a tracking point of view.

The second remark is related to the trade-off between minimizing phase error jitter and mean phase error, both ultimately leading, when becoming large, to unsafe tracking operations, as clearly emerging from Figure 2. For a given frequency rate, the loop parameters will be selected in order to contain both errors, according to Eqs. (1) to (4).

The final remark is that the specific characterization of Figure 2 has no general applicability, as it has been built upon results from a specific operational system, in a limited set of operating points. Aspects related to specific acquisition mechanisms and performance, non-ideal behaviors, or different operating points may drastically change or limit the useful region of operations. Furthermore, the requirements of minimizing the degradation from imperfect carrier synchronization onto the telemetry and ranging functions may be more restrictive than the requirement to preserve the lock condition during tracking. Therefore, a characterization of the selected operational system with respect to the specific mission requirements must be performed, at least equivalent to the one presented in sections 4 and 5.

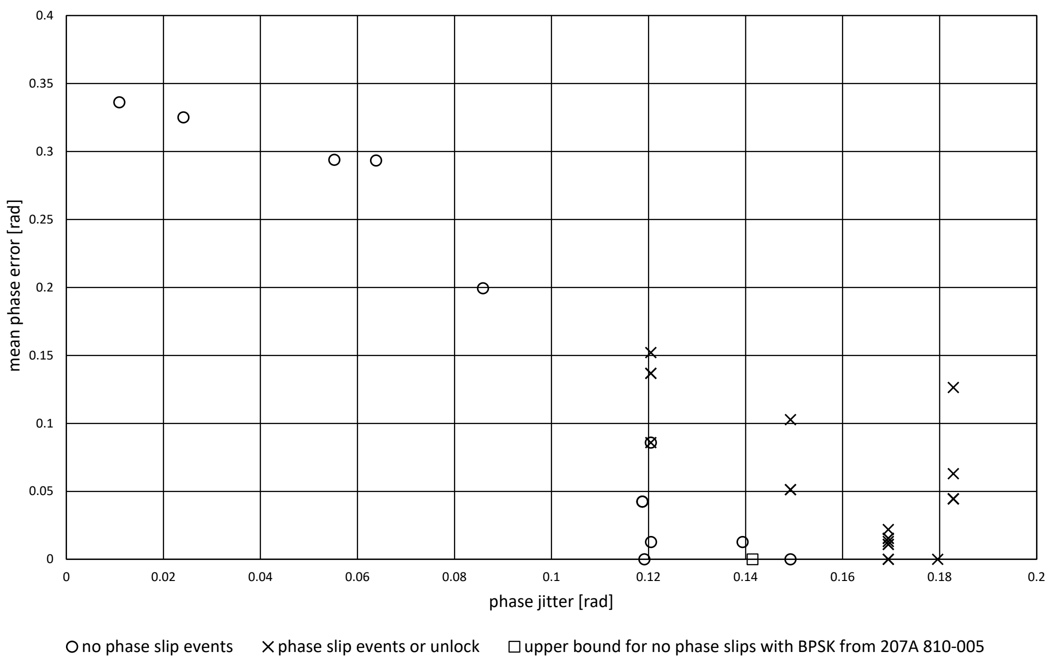

Figure 2: evaluation of carrier phase jitter and mean for the tested configurations, for reduced P_TT_s/N_0 in the range -5 to -3 dB. The product B_LT_s is in the range 10^{-5} to $2.3\cdot10^{-3}$ across the various tests.

Table 3: characterisation of phase slip events - test results for GMSK+PN with decision directed synchroniser

test case Id	TM mod.	PN ranging code and mod.	channel coding	reduced $P_T T_s/N_0$ [dB]	R_s [symbol/s]	PN Chip rate [chip/s]	B_{Lc} [Hz]	$2B_{L0}$ [Hz]	$B_{L0}T_s$	ζ_0	$2B_L$ [Hz]	$B_L T_s$	ζ	ω_n [1/s]	expected phase jitter [rad]	expected phase jitter [degrees]	frequency rate [Hz/s]	γ [rad]	expected mean phase error [rad]	expected mean phase error [degrees]	measured phase jitter [degrees]	measured mean phase error [degrees]	measured RG delay [ns]	measured RG delay jitter [ns]	duration [s]	Positive phase slip events	Negative phase slip events	Phase slip events
1 a				-3				1978	4.9E-03	1	908	2.3E-03	0.57	900	0.169	9.7	1415	0.011	0.011	0.6	**10.8**	**unmeasurable**	**323.1**	**6.3**	**1060**	**24**	**53**	**77**
1 b				-3				1978	4.9E-03	1	908	2.3E-03	0.57	900	0.169	9.7	-1415	0.011	0.011	0.6	**10.8**		**323.1**	**6.3**	**345**	**15**	**10**	**25**
2 a				-3				1978	4.9E-03	1	908	2.3E-03	0.57	900	0.169	9.7	2829	0.022	0.022	1.3	**10.9**	**0.9**	**323.4**	**7.1**	**168**	**no sustained lock**		
3 n				-3				1978	4.9E-03	1	908	2.3E-03	0.57	900	0.169	9.7	0	0.000	0.000	0.0	**10.9**		**322.8**	**6.2**	**229**	**8**	**6**	**14**
4 a				-3				1978	4.9E-03	1	908	2.3E-03	0.57	900	0.169	9.7	2000	0.015	0.015	0.9	**11.6**	**unmeasurable**	**322.6**	**6.6**	**1660**	**39**	**107**	**146**
4 b				-3				1978	4.9E-03	1	908	2.3E-03	0.57	900	0.169	9.7	-2000	0.015	0.015	0.9	**11.6**		**322.6**	**6.6**	**2963**	**163**	**55**	**218**
5 a				-3				1978	4.9E-03	1	908	2.3E-03	0.57	900	0.169	9.7	1700	0.013	0.013	0.8	**11.6**		**323.1**	**6.6**	**882**	**18**	**42**	**60**
6 a				-3				1000	2.5E-03	1	459	1.1E-03	0.57	455	0.120	6.9	2829	0.086	0.086	4.9	**7.2**	**4.6**	**323.0**	**6.2**	**2676**	**None**		
6 b				-3				1000	2.5E-03	1	459	1.1E-03	0.57	455	0.120	6.9	-2829	0.086	0.086	4.9	**7.2**	**4.6**	**323.0**	**6.2**	**2324**	**1**	**0**	**1**
7 a				-3				1000	2.5E-03	1	459	1.1E-03	0.57	455	0.120	6.9	4500	0.136	0.137	7.8	**7.3**	**7.9**	**322.7**	**6.0**	**970**	**0**	**5**	**5**
7 b				-3				1000	2.5E-03	1	459	1.1E-03	0.57	455	0.120	6.9	-4500	0.136	0.137	7.8	**7.3**	**7.9**	**322.7**	**6.0**	**1325**	**2**	**0**	**2**
8 a				-3				1000	2.5E-03	1	459	1.1E-03	0.57	455	0.120	6.9	5000	0.152	0.152	8.7	**no lock**							
9 a				-2.9				1000	2.5E-03	1	461	1.2E-03	0.57	457	0.119	6.8	1415	0.043	0.043	2.4	**7.3**	**2.4**	**322.8**	**6.2**	**5287**	**None**		
9 b	GMSK BT_s = 0.25	T4B, sin, m_{RG} = 0.444 rad	uncoded	-2.9	2.0E+05	206540	0.063	1000	2.5E-03	1	461	1.2E-03	0.57	457	0.119	6.8	-1415	0.043	0.043	2.4	**7.3**	**2.4**	**322.8**	**6.2**	**4689**	**None**		
10 a				-4				1000	2.5E-03	1	417	1.0E-03	0.52	416	0.149	8.6	1415	0.051	0.051	2.9	**8.89**	**3.0**	**323.3**	**7.5**	**1847**	**0**	**2**	**2**
10 b				-4				1000	2.5E-03	1	417	1.0E-03	0.52	416	0.149	8.6	-1415	0.051	0.051	2.9	**8.89**	**3.0**	**323.3**	**7.5**	**2141**	**4**	**0**	**4**
11 a				-4				1000	2.5E-03	1	417	1.0E-03	0.52	416	0.149	8.6	2830	0.103	0.103	5.9	**7.95**	**6.0**	**322.5**	**6.7**	**270**	**no sustained lock**		
12 a				-5.1				1000	2.5E-03	1	376	9.4E-04	0.47	376	0.183	10.5	1415	0.063	0.063	3.6	**11.17**	**4.0**	**323.7**	**9.7**	**637**	**1**	**22**	**23**
13 a				-5.1				1000	2.5E-03	1	376	9.4E-04	0.47	376	0.183	10.5	2830	0.126	0.126	7.2	**no lock**							
14 a1				-5.1				1000	2.5E-03	1	376	9.4E-04	0.47	376	0.183	10.5	1000	0.045	0.045	2.6	**11.07**	**2.7**	**322.5**	**9.2**	**1344**	**3**	**23**	**26**
14 b1				-5.1				1000	2.5E-03	1	376	9.4E-04	0.47	376	0.183	10.5	-1000	0.045	0.045	2.6	**11.07**		**323.9**	**9.0**	**701**	**9**	**0**	**9**
14 a2				-5.1				1000	2.5E-03	1	376	9.4E-04	0.47	376	0.183	10.5	1000	0.045	0.045	2.6	**11.07**		**amb. not res.**		**173**	**0**	**2**	**2**
14 b2				-5.1				1000	2.5E-03	1	376	9.4E-04	0.47	376	0.183	10.5	-1000	0.045	0.045	2.6	**11.07**		**321.7**	**9.3**	**1019**	**23**	**3**	**26**
15 n				-2.9				1000	2.5E-03	1	461	1.2E-03	0.57	457	0.119	6.8	0	0.000	0.000	0.0	**7.23**	**unmeasurable**	**322.0**	**6.3**	**1199**	**None**		
16 n				-4				1000	2.5E-03	1	417	1.0E-03	0.52	416	0.149	8.6	0	0.000	0.000	0.0	**8.74**		**322.5**	**7.7**	**1199**	**None**		
17 n				-5				1000	2.5E-03	1	380	9.5E-04	0.47	379	0.180	10.3	0	0.000	0.000	0.0	**10.7**		**323.4**	**8.2**	**1199**	**1**	**4**	**5**
18 n				-3				1978	4.9E-03	1	908	2.3E-03	0.57	900	0.169	9.7	0	0.000	0.000	0.0	**11.69**		**322.9**	**6.3**	**1199**	**42**	**41**	**83**

Table 4: maximum γ with high Doppler rate scenario - test results for GMSK+PN with decision directed synchroniser, with T2B (top) and T4B (bottom)

test case Id	TM mod.	PN ranging code and mod.	channel coding	reduced $P_T T_s/N_0$ [dB]	R_s [symbol/s]	PN Chip rate [chip/s]	B_{Lc} [Hz]	$2B_{L0}$ [Hz]	$B_{L0}T_s$	ζ_0	$2B_L$ [Hz]	$B_L T_s$	ζ	ω_n [1/s]	expected phase jitter [rad]	expected phase jitter [degrees]	frequency rate [Hz/s]	γ [rad]	expected mean phase error [rad]	expected mean phase error [degrees]	measured phase jitter [degrees]	measured mean phase error [degrees]	measured RG delay [ns]	measured RG delay jitter with no sweep [ns]	measured RG delay variation during sweep [ns]	duration [s]	lost TM frames	phase slip events
A2	GMSK $BT_s = 0.25$	T2B, sin, m_{RG} = 0.444 rad	turbo (k=8920, r=1/4)	-3.0	1.0E+06	1032700	0.4	1050	5.3E-04	1	482	2.4E-04	0.57	478	0.055	3.2	10535	0.290	0.294	16.8	**3.5**	**16.8**	**242.8**	**1.8**	**2.1**	**2181**	**0**	**None**
B2				-3.0	5.0E+06	5032700	0.4	1000	1.0E-04	1	459	4.6E-05	0.57	455	0.024	1.4	10535	0.319	0.325	18.6	**1.1**	**18.2**	**242.3**	**0.2**	**0.5**	**583**		
C2				-3.0	5.0E+05	502700	0.4	1270	1.3E-03	1	583	5.8E-04	0.57	578	0.086	4.9	10535	0.198	0.199	11.4	**4.9**	**11.7**	**241.9**	**5.9**	**5.9**	**585**		
D2				-3.0	2.4E+07	23997300	0.4	984	2.1E-05	1	452	9.4E-06	0.57	448	0.011	0.6	10535	0.330	0.336	19.3	**0.6**	**19.8**	**242.5**	**0.01**	**0.3**	**1279**		
E2				9.5	1.0E+06	1002700	0.4	530	2.7E-04	1	530	2.7E-04	1.00	424	0.006	0.3	10535	0.368	0.377	21.6	**0.3**	**21.0**	**242.4**	**0.2**	**0.8**	**540**		
F2				-3.0	1.0E+06	1002700	0.4	5000	2.5E-03	1	2295	1.1E-03	0.57	2276	0.120	6.9	10535	0.013	0.013	0.7	**7.8**	**not meas.**	**242.1**	**1.6**	**2.1**	**578**		
G2	GMSK $BT_s = 0.5$			9.7	1.0E+06	1002700	0.4	475	2.4E-04	1	475	2.4E-04	1.00	380	0.005	0.3	10535	0.458	0.476	27.3	**0.3**	**26.2**	**242.2**	**0.2**	**1.0**	**607**		
H2			turbo (k=8920, r=1/6)	-5.0	1.0E+06	1002700	0.4	1050	5.3E-04	1	483	2.4E-04	0.57	479	0.064	3.7	10535	0.289	0.293	16.8	**3.5**	**16.9**	**244.7**	**2.7**	**3.9**	**579**		
I2				-5.0	1.0E+06	1002700	0.4	5000	2.5E-03	1	2298	1.1E-03	0.57	2279	0.139	8.0	10535	0.013	0.013	0.7	**7.9**	**not meas.**	**244.0**	**3.2**	**3.3**	**606**		

test case Id	TM mod.	PN ranging code and mod.	channel coding	reduced $P_T T_s/N_0$ [dB]	R_s [symbol/s]	PN Chip rate [chip/s]	B_{Lc} [Hz]	$2B_{L0}$ [Hz]	$B_{L0}T_s$	ζ_0	$2B_L$ [Hz]	$B_L T_s$	ζ	ω_n [1/s]	expected phase jitter [rad]	expected phase jitter [degrees]	frequency rate [Hz/s]	γ [rad]	expected mean phase error [rad]	expected mean phase error [degrees]	measured phase jitter [degrees]	measured mean phase error [degrees]	measured RG delay [ns]	measured RG delay jitter with no sweep [ns]	measured RG delay variation during sweep [ns]	duration [s]	lost TM frames	phase slip events
A4	GMSK $BT_s = 0.25$	T4B, sin, m_{RG} = 0.444 rad	turbo (k=8920, r=1/4)	-3.0	1.0E+06	1032700	0.4	1050	5.3E-04	1	482	2.4E-04	0.57	478	0.055	3.2	10535	0.290	0.294	16.8	**3.0**	**16.5**	**308.4**	**1.3**	**1.5**	**287**	**0**	**None**
B4				-3.0	5.0E+06	5032700	0.4	1000	1.0E-04	1	459	4.6E-05	0.57	455	0.024	1.4	10535	0.319	0.325	18.6	**1.2**	**18.7**	**308.1**	**0.1**	**0.2**	**287**		
C4				-3.0	5.0E+05	502700	0.4	1270	1.3E-03	1	583	5.8E-04	0.57	578	0.086	4.9	10535	0.198	0.199	11.4	**5.1**	**12.1**	**309.6**	**3.9**	**3.8**	**191**		
D4				-3.0	2.4E+07	23997300	0.4	984	2.1E-05	1	452	9.4E-06	0.57	448	0.011	0.6	10535	0.330	0.336	19.3	**0.6**	**20.1**	**307.9**	**0.02**	**0.1**	**192**		
E4				9.0	1.0E+06	1002700	0.4	530	2.7E-04	1	530	2.7E-04	1.00	424	0.006	0.3	10535	0.368	0.377	21.6	**0.3**	**21.1**	**308.3**	**0.3**	**0.5**	**192**		
F4				-3.0	1.0E+06	1002700	0.4	5000	2.5E-03	1	2295	1.1E-03	0.57	2276	0.120	6.9	10535	0.013	0.013	0.7	**6.7**	**not meas.**	**308.3**	**0.8**	**1.4**	**190**		
G4	GMSK $BT_s = 0.5$			9.5	1.0E+06	1002700	0.4	475	2.4E-04	1	475	2.4E-04	1.00	380	0.005	0.3	10535	0.458	0.476	27.3	**0.2**	**26.2**	**308.0**	**0.3**	**0.6**	**192**		
H4			turbo (k=8920, r=1/6)	-5.0	1.0E+06	1002700	0.4	1050	5.3E-04	1	483	2.4E-04	0.57	479	0.064	3.7	10535	0.289	0.293	16.8	**3.7**	**18.0**	**308.9**	**2.0**	**2.2**	**192**		
I4				-5.0	1.0E+06	1002700	0.4	5000	2.5E-03	1	2298	1.1E-03	0.57	2279	0.139	8.0	10535	0.013	0.013	0.7	**8.2**	**not meas.**	**307.9**	**2.5**	**2.2**	**192**		

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