Proximity Link Throughput Enhancements via Raptor Code Technology

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Outline

Introduction

- Proximity links are designed for the purpose of data transfer among probes, landers, rovers, orbiting CubeSat constellations and CubeSat relays.
- Prox-1 standard based communications technology is used for reliable operation of proximity links.
- Current Prox-1 standard severely limits the throughput when the environment differs from the optimal operating point such as when the channel has higher/lower SNR.
- We aim to develop energy-efficient, high throughput and low latency communications technology for proximity links.

State-of-the-Art

Current Prox-1 Standard: CCSDS 211.0-B-5

- Data packets are transmitted as PLTUs, which can be up to 16384 bits long.
- Each PLTU is divided into several 1024-bit message blocks.
- A message block is encoded by a $R = 1/2$ LDPC code in the C&S sublayer. LDPC codewords pertaining to a PLTU are transmitted.
- A Go-Back-N ARQ protocol operating on the PLTU level is present in the data link layer (DLL).
- Performance of the $R = 1/2$ LDPC code and the Go-Back-N ARQ protocol determine the throughput.

CCSDS 211.0-B-5 Simplified Model

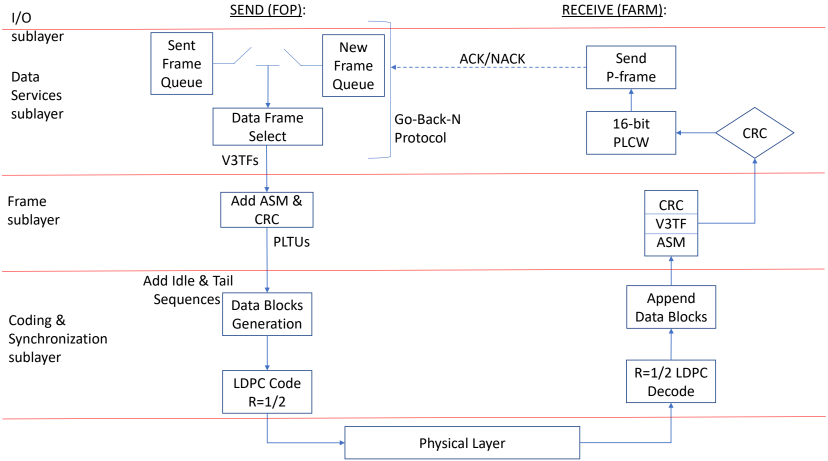


Figure: Simplified layered model of the DLL of CCSDS 211.0-B-5 Proximity-1 standard.

Proposed Rateless Code Model

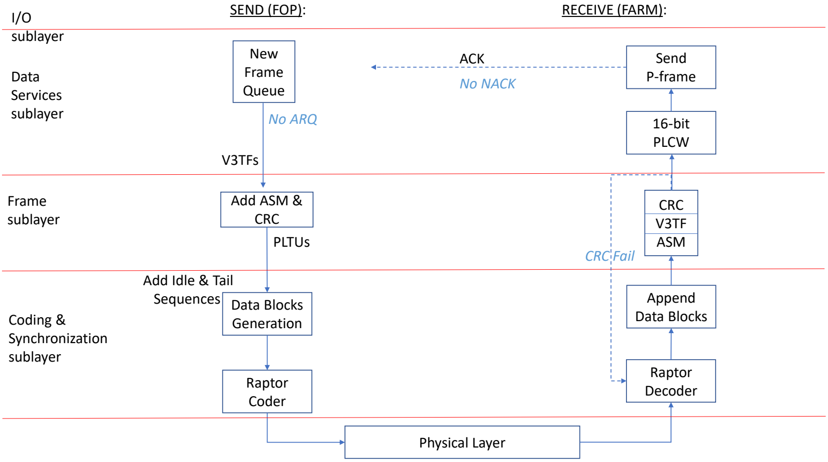


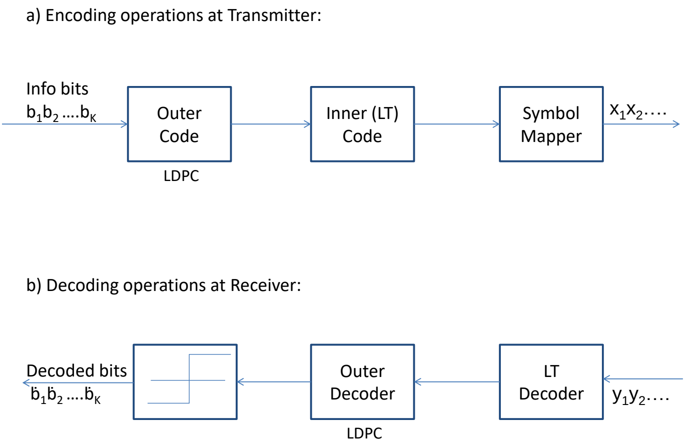
Figure: Simplified layered model of the DLL of the Prox-1 standard with Raptor code.

Rateless Code Approach

- Approach is based on **rateless codes over the AWGN channel**.
- Raptor code is one form of rateless code. Message blocks of a PLTU are Raptor encoded.
- Parity symbols for a Raptor encoded message block are incrementally transmitted with a variable code-rate until the receiver decoding succeeds.
- All the message blocks and thus, the PLTUs can be successfully transmitted by variable-rate transmission.
- **Ideally**, there is no need for a PLTU level ARQ protocol in the DLL when C&S sublayer uses Raptor codes.

New Design

Figure: Block diagram view of the Raptor encoder and decoder.



Outer code

- High rate **LDPC**: Clean up of inner code error-floor.
- Code-rate $R = \frac{K}{N} = 0.95$. Regular design, $d_v = 4$ and $d_c = 80$.
- Parity Check Matrix: $H \triangleq [Q|P]$.
  $\mathbf{H} : (N-K)*N$; $\mathbf{Q} : (N-K)*K$; $\mathbf{P} : (N-K)*(N-K)$

$$Q = \begin{bmatrix} 1 & 1 & \dots & 1 \\ 1 & 1 & \dots & 1 \\ 1 & 1 & \dots & 1 \\ 1 & 1 & \dots & 1 \end{bmatrix} \tag{1}$$

1. Q is initially a $4 * 76$ matrix of all ones.
2. Each 1 is replaced by a π_i, a random permutation block of size $M * M$, where $M = \frac{N-K}{4} = \frac{K}{4} \cdot \frac{1}{19}$.
3. Then Q expands to $(N-K) * K$.

LDPC Encoder

- Design of P: Initially a $4 * 4$ matrix of all ones. Each 1 is replaced by a π_i.

$$P = \begin{bmatrix} \pi_1 & \pi_2 & \pi_3 & \pi_4 \\ \pi_5^o & \pi_6 & \pi_7 & \pi_8 \\ \pi_9 & \pi_{10}^o & \pi_{11} & \pi_{12} \\ \pi_{13} & \pi_{14} & \pi_{15}^o & \pi_{16} \end{bmatrix} \tag{2}$$

  π_i^o is the same as π_i with one column replaced by an all-zero column.

- Construct P s.t. no two rows of P add up to an all-zero row, i.e., P is invertible in GF(2). From H, the generator matrix G_o is constructed.

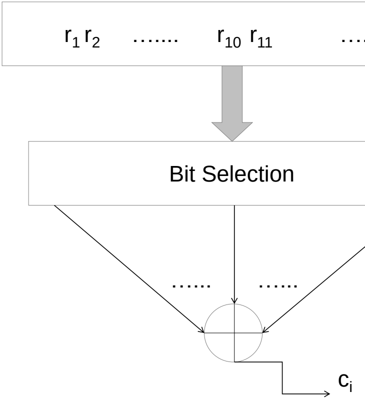
- Codeword $\mathbf{c}$ is given by

$$\mathbf{c} = \mathbf{b} G_o G_{lt}, \tag{3}$$

  G_o and G_{lt} are the LDPC and LT code generator matrices.

LT Encoder

Figure: K information bits. Uniformly select d out of K bits, XOR operation on them $\rightarrow$ parity symbol. Repeat as-needed.



LT Encoder

- Each parity/code bit of LT code has a degree d. *Two* approaches are considered.
- **Approach 1**: Each code bit c_i has a degree d_i that is random and sampled according to a degree distribution $\Omega(x)$.

$$\Omega(x) = .00477x^1 + .26101x^2 + .0924x^3 + .06913x^8 + .51223x^9 + .06046x^{60} \tag{4}$$

- Since the Prox-1 standard has $R = 1/2$ LDPC code, the degree distribution $\Omega(x)$ has been optimized for rate $R = 1/2$.
- $\Omega(x)$ minimizes the bit error rate (BER) at code-rate $R = 1/2$ and yet can generate an infinite number of parity bits.

LT Encoder

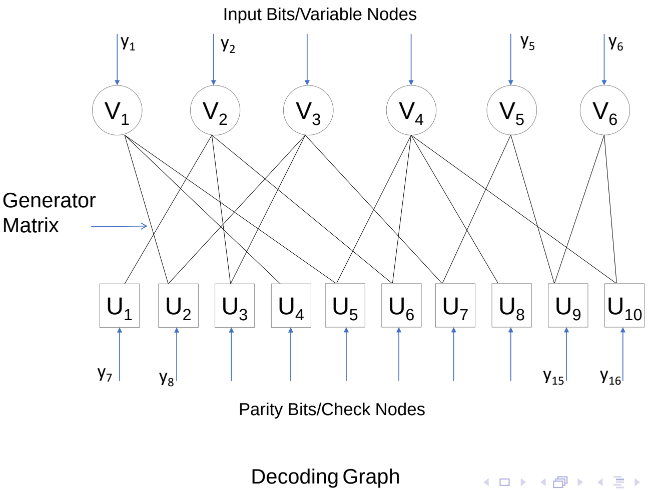
- **Approach 2**: The parity bits are assigned according to a structure.
- The first group of parity bits are assigned degree d_1, the second group of parity bits are assigned degree d_2 and so on.
- The degrees are in descending order, i.e., $d_1 \geq d_2 \geq d_3 \geq d_4 \cdots$.
- The number of parity bits in group $\#i$ is L_i and they are adjacent to one another within each group.

Table: Descending order degrees (DoD) approach to generate the LT code bits. Note N is the number of intermediate code bits output by the R=0.95 LDPC outer code.

Systematic	$d_1 = 12$	$d_2 = 6$	$d_3 = 4$	$d_4 = 2$
N	$L_1 = \alpha_1 N$	$L_2 = \alpha_2 N$	$L_3 = \alpha_3 N$	$L_4 = \alpha_4 N$
	$\alpha_1 = .27$	$\alpha_2 = .146$	$\alpha_3 = 1.39$	$\alpha_4 = \infty$

LT Decoder

Figure: Belief Propagation (BP) decoding for LT codes.



BP Decoder

1. Variable Nodes: K

$$L_{v_i c_j} = L_{v_i} + \sum_{N_v(i)\backslash j} L_{c_{j'} v_i}. \tag{5}$$

2. Check Nodes: N_{It}

$$L_{c_j v_i} = 2\ atanh\left[tanh\left(\frac{L_{c_j}}{2}\right) \prod_{N_c(j)\backslash i} tanh\left(\frac{L_{v_{i'} c_j}}{2}\right)\right]. \tag{6}$$

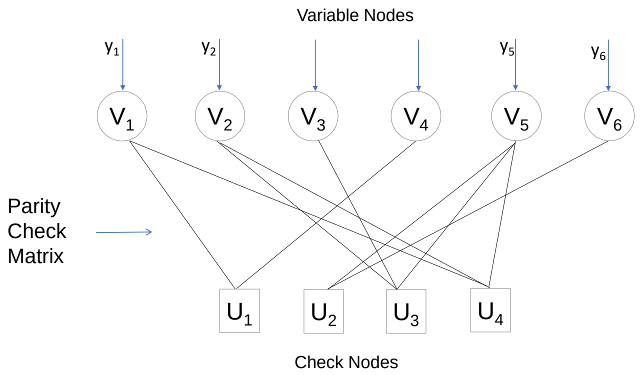
3. Output:

$$\underbrace{L^o_{v_i}}_{APP\ \ LLR} = \underbrace{L_{v_i}}_{Channel\ \ LLR} + \underbrace{\sum_{N_v(i)} L_{c_{j'} v_i}}_{Code\ \ LLR}. \tag{7}$$

4.
   - $N_v(i)$: Rows of Generator Matrix G.
   - $N_c(j)$: Col's of Generator Matrix G. *Indices of* 1*'s*.

LDPC Decoder

Figure: Belief Propagation decoding for LDPC codes.



Tanner Graph for R=1/3 LDPC code

BP Decoder

1. Variable Nodes: N

$$L_{v_i c_j} = L_{v_i} + \sum_{N_v(i)\setminus j} L_{c_{j'} v_i}. \tag{8}$$

2. Check Nodes: $N - K$

$$L_{c_j v_i} = 2\ atanh\Big[\prod_{N_c(j)\setminus i} tanh\left(\frac{L_{v_{i'} c_j}}{2}\right)\Big]. \tag{9}$$

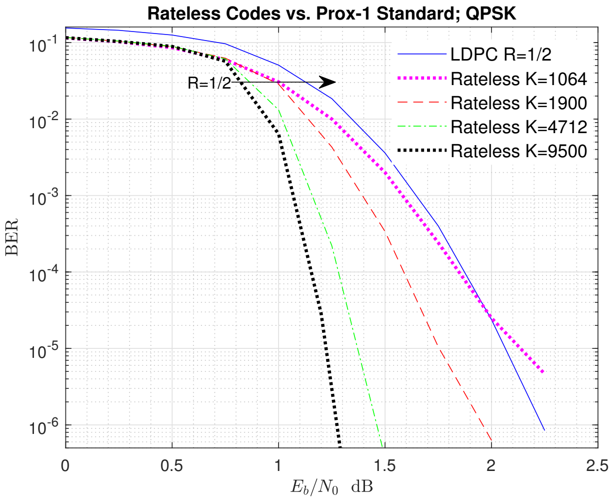
3. Output:

$$L^o_{v_i} = L_{v_i} + \sum_{N_v(i)} L_{c_{j'} v_i}. \tag{10}$$

4.
   - $N_v(i)$: Col's of Parity Check Matrix H.
   - $N_c(j)$: Rows of Parity Check Matrix H.

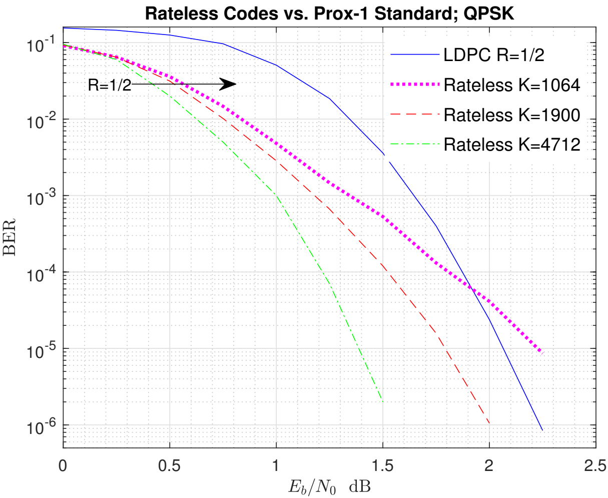
BER Plot

Figure: BER comparison between Raptor codes (Approach 1) and Prox-1 LDPC.



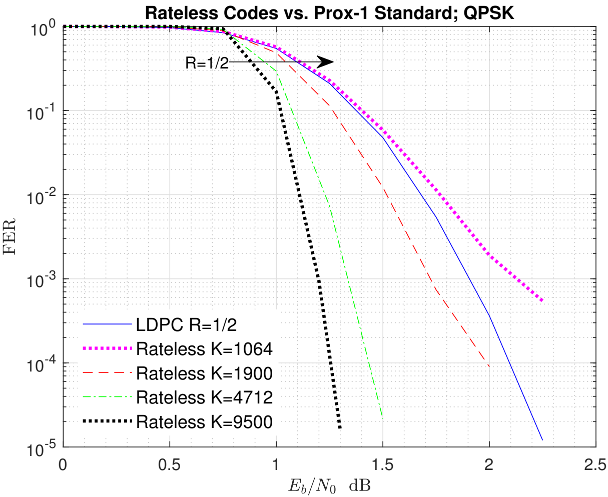
BER Plot

Figure: BER comparison between Raptor codes (Approach 2) and Prox-1 LDPC.



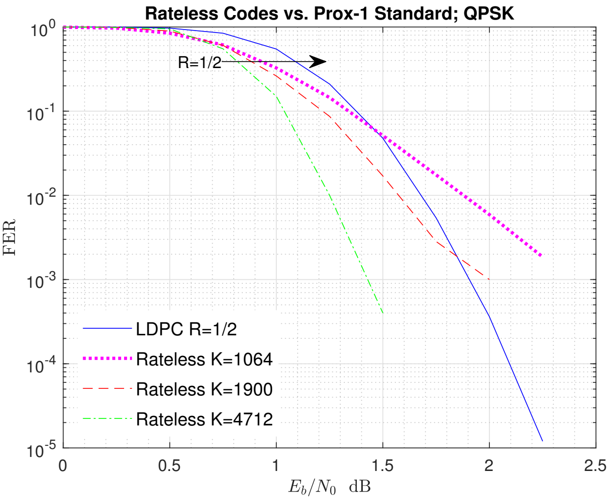
FER Plot

Figure: FER comparison between Raptor codes (Approach 1) and Prox-1 LDPC.



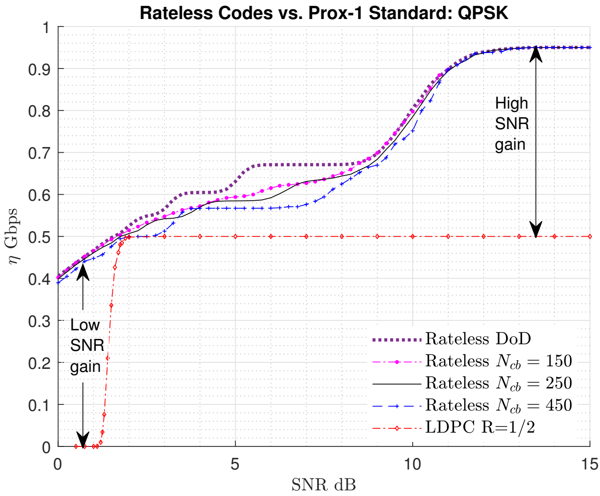
FER Plot

Figure: FER comparison between Raptor codes (Approach 2) and Prox-1 LDPC.



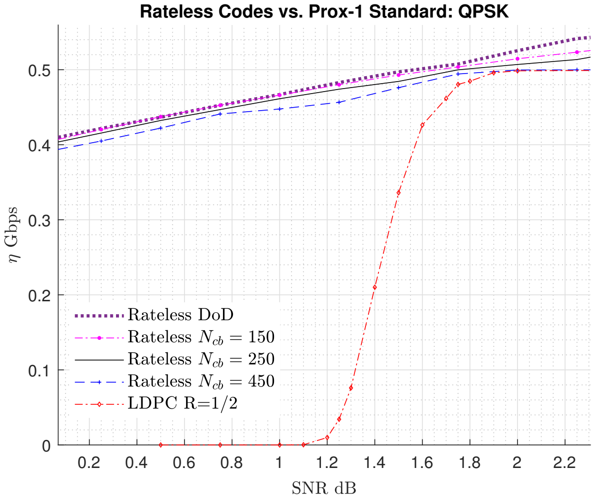
Throughput

Figure: A plot of throughput $\eta = \mathbb{E}[\frac{K}{T}]$ versus SNR with a 1 GHz clock.



Throughput

Figure: Throughput η comparison in the low SNR regime with a 1 GHz clock.



Throughput

1. For the Prox-1 system, the optimal SNR is 2 dB.
2. In both low and high SNR, Raptor system has throughput gain over Prox-1.
3. Low SNR regime: Presence of intermittent links
   - Prox-1 LDPC has zero throughput.
   - Raptor has 2x to 10x gain.
4. High SNR regime:
   - 30% gain at 2dB above optimal.
   - 90% gain at 8dB above optimal.

Computational Costs

- From a processor/FPGA perspective, the computational costs translate to the processing speed and power consumption of encoders and decoders.
- Prox-1 LDPC encoder: The codeword c is given by

$$c = bG_1, \tag{11}$$

  where G_1 is the generator matrix with dimensions $\dim G_1 = 1024 * 2560$.
- The generator matrix G_1 has the structure

$$G_1 \triangleq [I|Q_1]. \tag{12}$$

- Q_1 is a *non-sparse matrix* with $\dim Q_1 = 1024 * 1536$ with column weights w_c

$$501 \leq w_c \leq 537. \tag{13}$$

Computational Costs

- Raptor Option: The codeword c is given by

$$c = bG_o G_{lt}. \tag{14}$$

- The outer code generator matrix $G_o \triangleq [I|Q_o]$ has dimensions $\dim G_o = 1064 * (1064 + 56)$. Note that $\underline{\dim Q_o = 1064 * 56}$.

- The LT code generator matrix $G_{lt} \triangleq [I|Q_{lt}]$ has dimensions $\dim G_{lt} = 1120 * (1120 + N_{lt})$. Note that Q_{lt} is a *low density (sparse) matrix*.

- The column weights w_c of matrix Q_{lt} satisfy

$$\begin{cases} 2 \leq w_c \leq 9, & \textit{Approach } 1 \\ 2 \leq w_c \leq 12, & \textit{Approach } 2. \end{cases} \tag{15}$$

Decoder Costs

Table: Number of operations for a single iteration of the SPA. $\mathbf{Nadd_{vc}}$ captures the number of additions in (5) and (8). The operations in (6)-(10) are similarly captured.

Property	Prox-1 LDPC	Outer LDPC	Approach 1	Approach 2
Blocklength K	1024	1064	1120	1120
$Nadd_{vc}$	17290	10080	22466	15568
$Nmul_{cv}$	7680	4480	11793	8344
$Ndiv_{cv}$	7680	4480	10673	7224
$Natan_{cv}$	7680	4480	10673	7224
$Nadd_{v}$	10240	5600	11793	8344
Retransmissions	*Linear* scaling	N/A	Incremental	Incremental

- Low SNR - Big difference between Prox-1 LDPC and Raptor option.
- High SNR - Both techniques have similar costs.

Conclusion

- Using rateless codes over the AWGN channel as an additional option for the Prox-1 standard can increase the throughput of proximity links.
- Let "CPL" be an acronym for denoting the costs, power consumption and latency for the encoding and decoding of V3TFs.
- The key takeaway messages on throughput and computational costs are summarized in the following table.
- New designs for both the Raptor encoder and decoder based on deep learning/AI enhancements, structured permutations, and sparse linear system algorithms are possible.

Conclusion

Table: Throughput and CPL comparison between the Prox-1 standard and the Raptor addition. LDGM stands for low density generator matrix.

Prox-1 Standard:

Regime	**Throughput**	**Encoder CPL**	**Decoder CPL**
Low SNR	Near-zero	***High***: Dense matrix multiplication	***High***: Linear scaling with retransmissions
High SNR	Good	***High***: Dense matrix multiplication	***Moderate***

Raptor Addition:

Regime	**Throughput**	**Encoder CPL**	**Decoder CPL**
Low SNR	2x to 10x gain over Prox-1	***Low***: LDGM multiplication	***Incremental***: Fresh decoding with new parity
High SNR	30-90% gain over Prox-1	***Low***: LDGM multiplication	*Approach 1*: ***High*** *Approach 2*: ***Moderate***

Conclusion

For future direction in terms of the CCSDS standards path or technology demonstration payload, the issues that need to be addressed are:

1. **Rateless Protocol**: Integrating the Raptor encoder and decoder designs into the layered protocol stack, i.e., DLL or higher layers of the Prox-1 standard.
2. Practical feedback link issues such as “factoring in long feedback delay”, “errors on the feedback link” and solutions for them.
    - Multiplex L codewords to solve large RTD in proximity links.

References

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Further Discussions

For further discussions on the topic and the future direction of technology development project, please feel free to contact

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