Consultative Committee on Space Data Systems Space Link Services

Coding & Synchronization Working Group

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CCSDS Turbo codes Channel interleaver. Design and Performance results with tumbling spacecraft and solar scintillation channel models

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Abstract

In CCSDS fall meeting 2017, JPL presented the Stereo-B anomaly [RD-1], which is a practical example of a "tumbling spacecraft".
This anomaly put in evidence that the Turbo decoders inherit a vulnerability to burst errors from their constituent convolutional codes. This vulnerability is observed in spacecraft operations, and in simulation, and understood through analysis.
In a previous contribution [RD-2] we proposed another channel model for considering other physical phenomena affecting the transmission like the fading induced by the solar scintillation.
A well know potential solution to such impairments, also proposed in [RD-1], is to place a channel interleaver immediately following the turbo encoder. The channel interleaver converts a bursty channel into an approximately IID channel (fast fading channel). This in turn eliminates the decoder's vulnerability to bursts.
In this contribution we provide performance results and design conclusions for the row-column interleaver and golden angle interleaver on the two considered channel models.

Introduction

Tumbling spacecraft

In Figure 1 and Figure 2 we show two examples of the oscillations of the received SNR taken from [RD-1]. They were used as reference for the introduction of the tumbling spacecraft model.

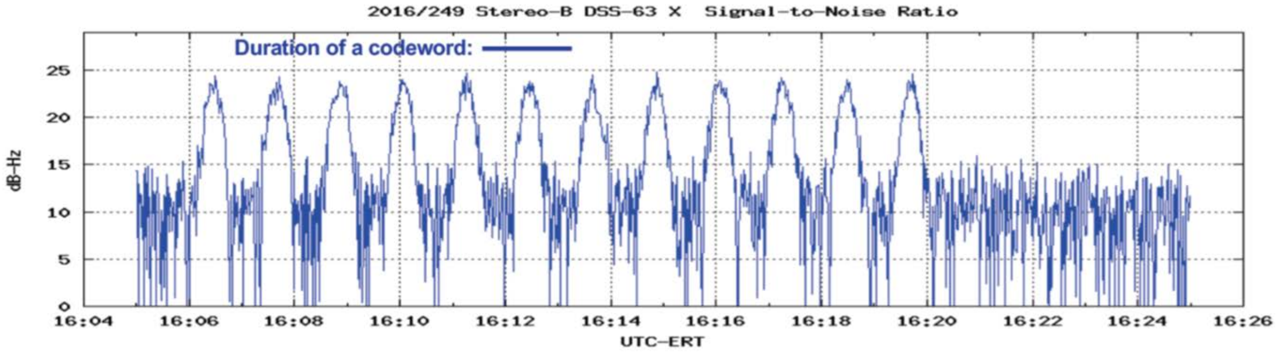

FIGURE 1. FIRST EXAMPLE OF OSCILLATIONS OF SNR WITH TUMBLING SPACECRAFT

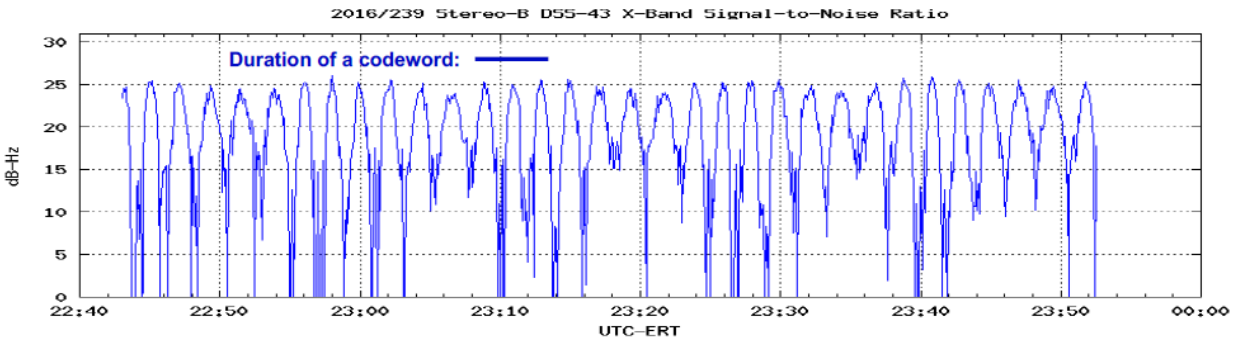

FIGURE 2 SECOND EXAMPLE OF OSCILLATIONS OF SNR WITH TUMBLING SPACECRAFT

The two examples show an approximately sinusoidal and deterministic behavior of the SNR (in dB). A suitable general model for such behavior is:

$$SNR(t) = SNR_0 + A\sin\left(\phi + 2\pi t\frac{\alpha}{T_f}\right)\ [dB],$$

where A is the SNR oscillations amplitude in dB, α is the frequency of the oscillations, normalized to the codeword rate, T_f is the codeword duration , and ϕ is an arbitrary phase offset.

As an example, for Figure 1, $A \approx 7$ dB and $\alpha \approx 1$, whereas for Figure 2, $A \approx 5$ dB and $\alpha \approx 3$.

1.1 Performance results

It appears that the value α in a general scenario can vary in a range that depends on the relative values of the baud-rate and the oscillation frequency of spacecraft. In the following, the performances are reported as a function of the value of α for three considered interleaver choices: no interleaver, golden angle interleaver, and row-column interleaver with properly tuned depth.

Two criteria have been considered for designing the row-column (write rows, read columns) interleaver depth (size $N = \frac{K+4}{R_c} = N_R \times N_C$).

In the first one, the interleaver maximizes the spreading for the set of bits generated by p consecutive steps of the constituent convolutional codes. This translates into a constraint on the number of columns of the RC interleaver $N_c = \frac{p}{R_c}$. Setting p=4, we obtain $N_c = \{8,12,16,24\}$ for the four rates, which are factors of N for all values of K.

In a second one the depth of the interleaver (N_R) varies depending on the value of α. We first compute the integer

$$N_R' = \frac{NR_c}{p\alpha},$$

with the rationale of associating to the set of bits generated by p consecutive steps of the constituent convolutional codes the SNR corresponding to a full oscillation period. We then pick for N_R the largest value smaller or equal than N_R' which is a factor of N.

For performance evaluation we considered as key performance indicator the SNR threshold to achieve a target **FER = 1E-3**. Performances of the considered solutions are reported in Figure 3 and Figure 4. We show results only relative to the two case $R_c = 1/6$ and $K = 1784$, and $R_c = 1/2$ and $K = 8920$.

The reported "SNR thresholds" in the figures report the estimated value of SNR_0 in the reported model to achieve the target FER. *Notice however that SNR_0 does not corresponds to the linear average SNR, as the model is sinusoidal in dB.*

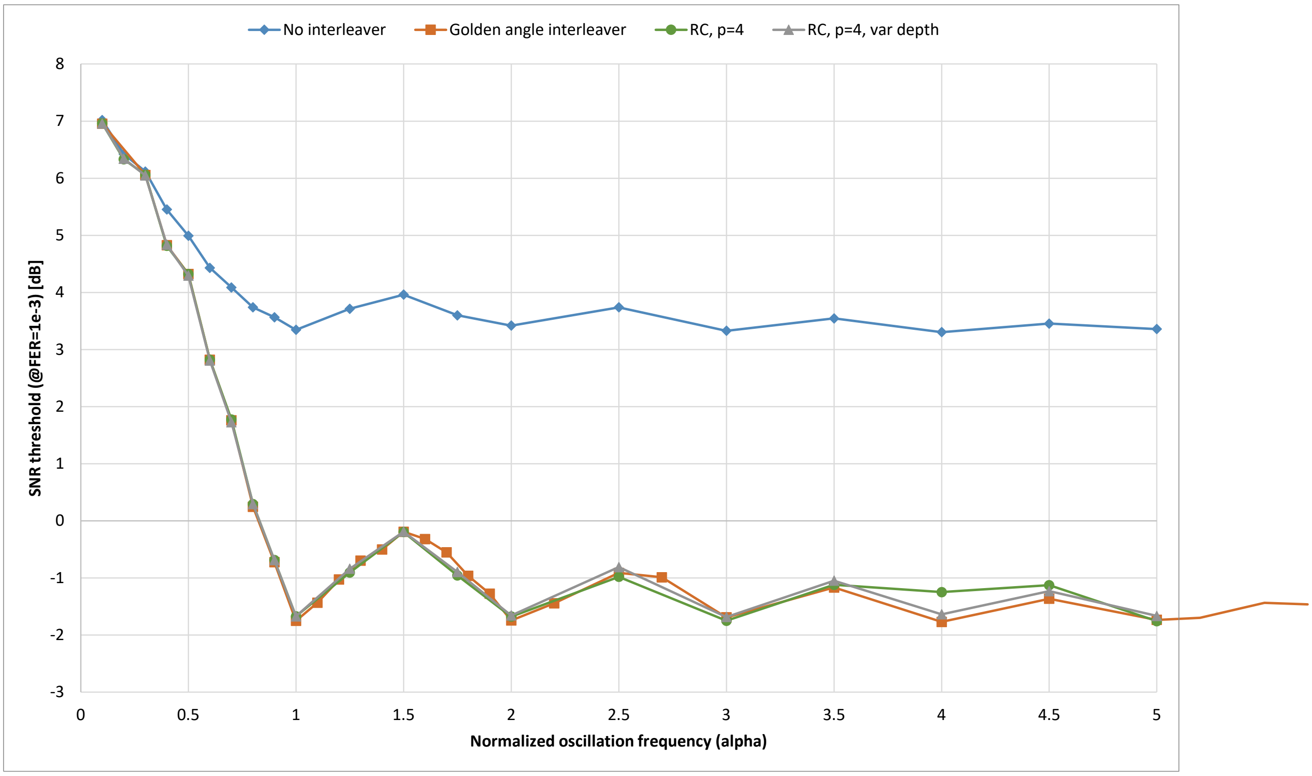

FIGURE 3. SNR THRESHOLDS (SNR_0) VS RELATIVE PERIODICITY OF TUMBLING SPACECRAFT. R_c=1/6, SHORT CODEWORDS, A=7.

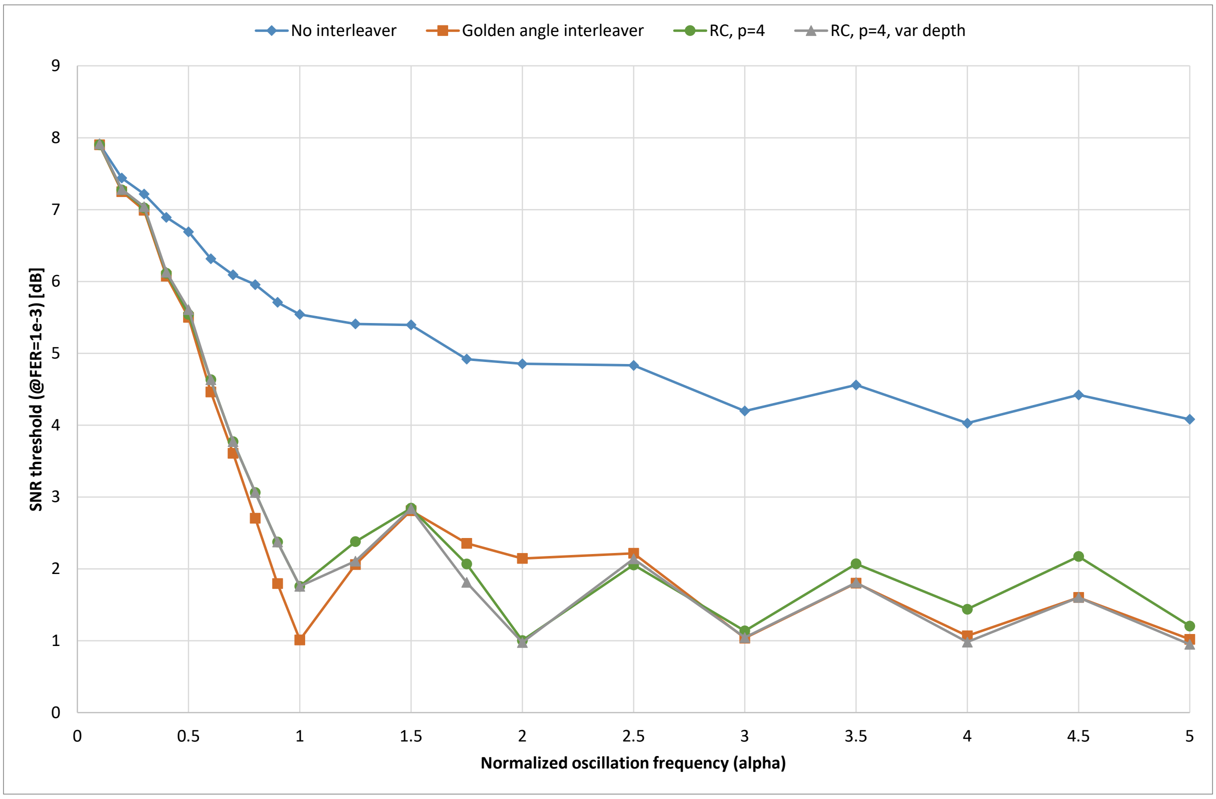

FIGURE 4. SNR THRESHOLDS (SNR_0) VS RELATIVE PERIODICITY OF TUMBLING SPACECRAFT. RC=1/2, LONG CODEWORDS, A=7.

The behavior of the performance curves can be interpreted considering that the FER performance of good codes is related to the accumulated “information density” of the channel. This random variable is approximately related to the SNR as follows.

$$MI_k \approx \eta_k + \frac{1}{T_f} \int_{kT_f}^{(k+1)T_f} \log_2\big(1 + SNR(t)\big)\, dt \,,$$

where η_k is a random variable and $FER_k \approx P[MI_k < R_c m]$[1].

Some comments are in order:

- **When the normalized periodicity α is smaller than one**, we observe a progressive degradation of the performance, since accumulated MI in each codeword is time variant. The performance is dominated by the worst-case SNR and ultimately the threshold loss depends on the worst SNR condition.
- In this situation all solutions based on *intra-frame interleaver* become not effective. <u>The adoption of inter-frame interleaver is to be considered if this range of values, corresponding to high baud rates and/or low oscillation frequencies, is of interest.</u>
- **When the normalized periodicity is larger than one,** we observe an attenuated oscillating behavior.
  - Minima are observed at integer values of α. This is the case where the statistic of MI_k is not dependent on k
  - When α assumes fractional values the SNR threshold degrades because also in this case the accumulated MI in each codeword is time variant and performance is dominated by the worst case.
- In this situation the adoption of an intra-frame interleaver yields large performance improvements, in the order of 5 dB for Figure 3, and 3-4 dB for Figure 4
- There are some small performance differences between the golden angle interleaver and considered row-column interleavers. There is not a clear optimal solution.

The usefulness of the addition of an intra-frame interleaver is evident.

The main question is then about the usefulness of an additional inter-frame interleaver, with depth D codewords, to be added on top of the intra-frame interleaver. This will be considered in the next section.

Solar scintillation

In [Rd-5] we proposed a more classical non-deterministic fading channel for assessing the interleaver performance in the presence of solar scintillation phenomena.

The received samples are given by:

$$r_k = a_k s_k + n_k,$$

[1] These expressions, which contain several approximations, are introduced only for the purpose of qualitatively comment the performance curve behavior.

where the discrete model for the complex gain sequence a_k is a Gaussian correlated stationary process with given spectrum and possibly nonzero mean values, yielding a Rician first order statistic.

We proposed to characterize the complex Gaussian process a_k with a single parameter spectrum depending on the coherence time (or Doppler spread), e.g.

$$S_a(f) \propto \frac{1}{1+\left(\frac{f}{f_0}\right)^2} = \frac{1}{1+(T_c R_s f)^2}$$

The effect of a multiplicative random process of coherence time M_C, measured in symbol interval units is that of reducing by a factor M_C the "*effective length*" of the adopted code.

The dimensioning of a suitable inter-frame interleaver in this case is related to the codeword duration M_E on one side and the channel memory M_C on the other side. The optimum interleaver size N should be the product of the two $N \propto M_E \times M_C$. Row-column interleavers, in both block or convolutional form, are good candidates in this scenario.

The effect of using an inter-frame row-column interleaver with size $D \times M_E$, with $D < M_c$, is that of reducing the channel memory (coherence time) by a factor *D.*

$$M_c' = \frac{M_c}{D}$$

The fine-tuning of the interleaver dimension as well as the design of the interleaver law can be performed only with a proper model of the first and second order channel statistics and the knowledge about the symbol duration (baud-rate).

1.2 Performance results

The assessment of the performance of system has been done by computing the SNR thresholds (in this case at **FER=1e-2**) versus the channel parameters $f_0 = T_s/T_c$ and the Rician factor K.

The results for the system *without channel interleaver* are reported in Figure 5, Figure 6, Figure 7, and Figure 8, for the four rates and K=1784. Each figure reports the SNR threshold versus the ratio T_c/T_s for two or three values of the Rician factors (K=-10, 0 and 10 dB).

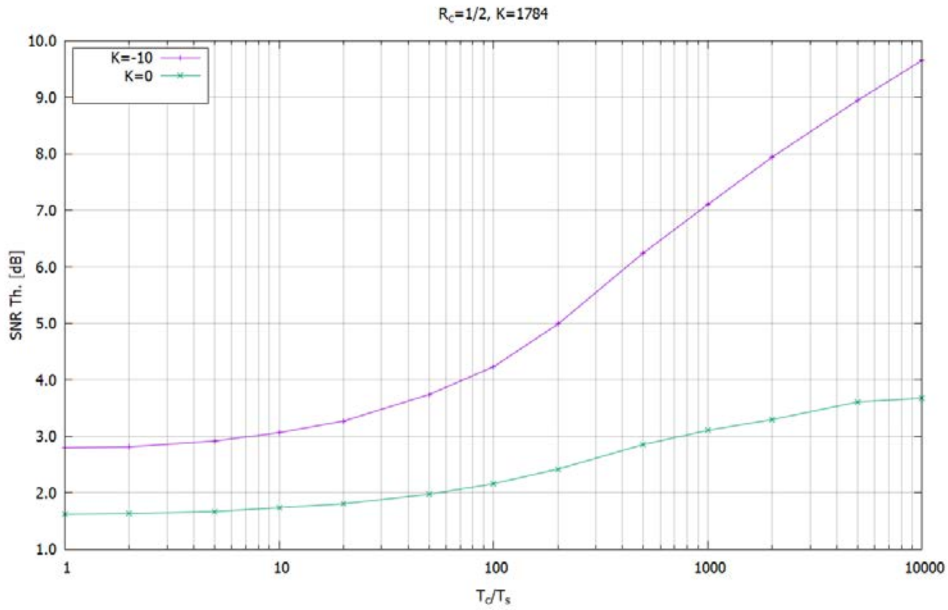

FIGURE 5. CCSDS CODE RATE 1/2, K=1784. SNR THRESHOLD (@FER=1E-2) FOR DIFFERENT VALUES OF THE RATIO T_c/T_s AND TWO VALUES OF THE RICE FACTOR K. NO INTERLEAVER.

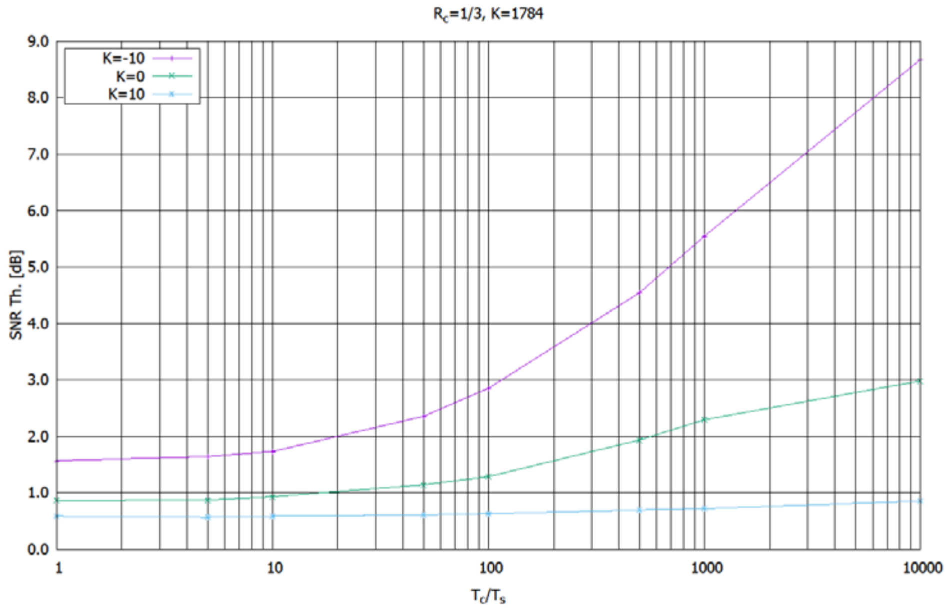

FIGURE 6. CCSDS CODE RATE 1/3, K=1784. SNR THRESHOLD (@FER=1E-2) FOR DIFFERENT VALUES OF THE RATIO T_c/T_s AND THREE VALUES OF THE RICE FACTOR K.

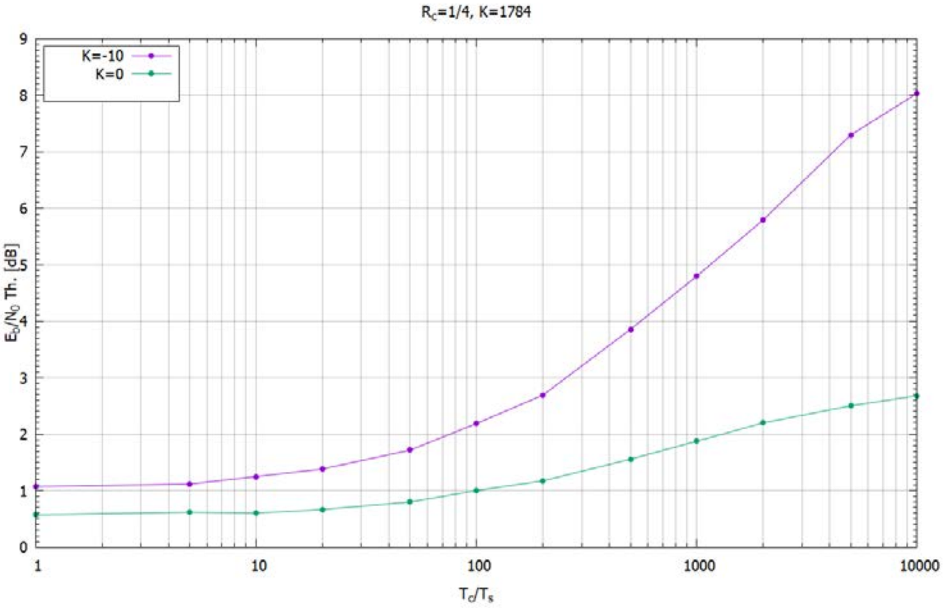

FIGURE 7 CCSDS CODE RATE 1/4, K=1784. SNR THRESHOLD FOR DIFFERENT VALUES OF THE RATIO T_c/T_s AND TWO VALUES OF THE RICE FACTOR K.

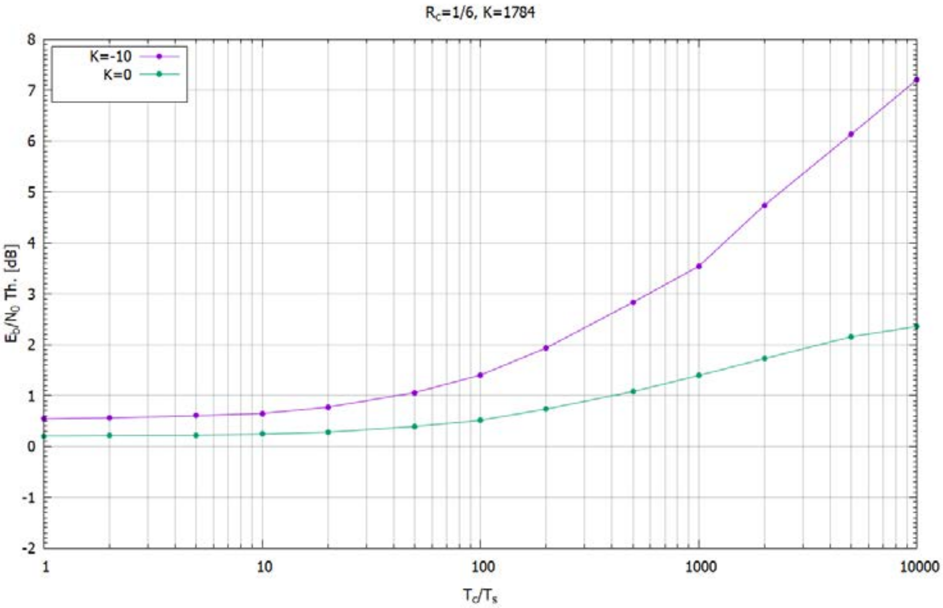

FIGURE 8. CCSDS CODE RATE 1/6, K=1784. DEGRADATION OF SNR THRESHOLD FOR DIFFERENT VALUES OF THE RATIO T_c/T_s AND TWO VALUES OF THE RICE FACTOR K.

As expected, the plots show large and unacceptable performance degradations for large values of the normalized coherence time when the Rician factor is small (0 or -10), but negligible when it is large.

In the solar scintillation model introduced in the [RD-4][RD-5], both coherence time and Rician factor are related to the scintillation index *m*. The measured relationship with the coherence time is reported in Figure 1 of [RD-2], while the Rician factor can be obtained as:

$$K = \frac{\sqrt{1-m^2}}{1-\sqrt{1-m^2}}$$

Thus, when increasing the scintillation index both the coherence time and the Rician factor increase, yielding opposite effect on the performance.

To jointly consider these effects in Figure 9 we report the SNR thresholds versus the *scintillation index*. The black curve reports the values of normalized coherence time T_c/T_s (right vertical axis). The corresponding Rician factor is indicated in the label. The considered symbol rate in this case is $R_s = 10$ kbaud.

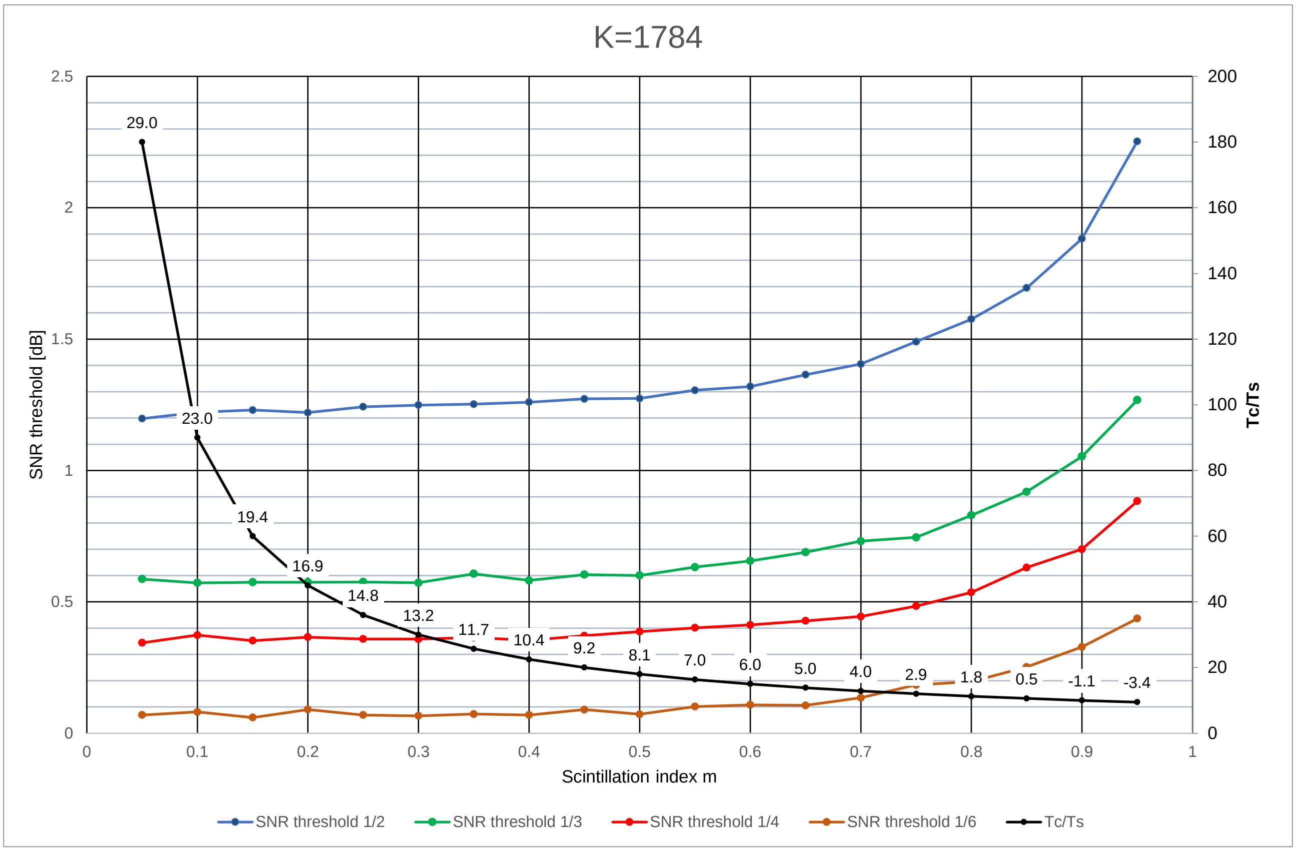

FIGURE 9. SNR THRESHOLDS FOR DIFFERENT VALUES OF THE SCINTILLATION INDEX M. BOTH RICIAN FACTOR AND COHERENCE TIME VARIES AS FUNCTION OF M. BLACK CURVE REPORTS VALUES OF COHERENCE TIME (RIGHT VERTICAL AXIS), WHILE THE CORRESPONDING RICIAN FACTOR IS REPORTED IN THE LABEL. THE BAUD RATE IS RS=10 KBAUD

The curves show that the performance degradation increases for larger scintillation indexes, that is for smaller values of the coherence time and smaller Rician factors.

As an example, with *m*=0.95, the Rician factor is *K*=-3.4 dB, and the coherence time around 10. The SNR threshold degradation is 0.4 dB for rate 1/6, 0.6 dB for rate 1/4, 0.7 dB for rate 1/3 and 1.0 dB for rate ½.

The degradation is due to the transition from the performance associated to the AWGN channel (high values of K) and that associated to the Rayleigh Fading channel (low values of K).

In order to assess the impact on performances of the insertion of an interleaver, we focus on the case with highest loss, with R=1/2, K=1784. In Figure 10 we show the SNR thresholds vs the baud rate for two high values of the scintillation index, corresponding to low Rician factors.

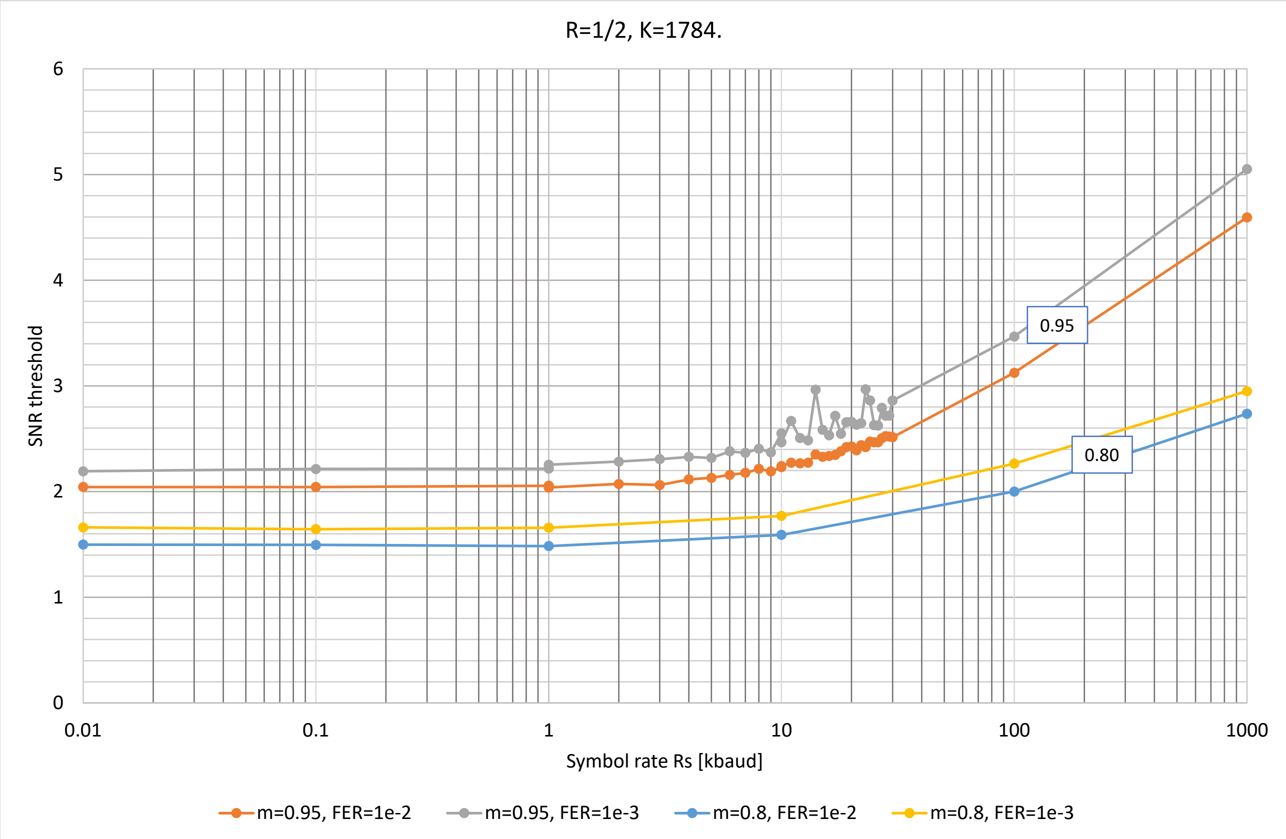

FIGURE 10. SNR THRESHOLDS VS BAUD RATE FOR M=0.80 AND M=0.95. RC=1/2, K=1784

Increasing the baud rate increases the normalized coherence time $M_c = R_s T_c$ and consequently reduces the effective codelength. The reduction of the effective codelength in turns degrades the slope of the FER performance curve and consequently the threshold. The figure shows that performance degradation occurs only when $R_s > 10$ kbaud.

Considering that the interleaver depth D has the effect of reducing the normalized coherence time the plot allows to quantify the threshold gain introduced by the insertion of an inter-frame interleaver. The gain is obtained by comparing the threshold at a given rate R_s with that at R_s/D

For example, an interleaver depth D=10, with scintillation index 0.95, yields a 1.4 dB gain at baud rate 1Mbaud, 0.9 dB at 100 kbaud, **0.2 dB at 10 kbaud and negligible gain at symbol rate below 1kbaud**. These gains would increase considering smaller FER values for the SNR threshold definition.

Considering that the typical mission baud rates are below 10kbaud and that the considered case is the worst case (small block size, large code rate and large scintillation index), the adoption of an inter-frame interleaver is then considered not necessary.

Conclusions

To summarize, the recommendation of this contribution are the following

- **The adoption of an intra-frame interleaver is strongly recommended**. It has no impact of memory and latency of the system and provides vary large performance improvements in some realistic scenario.
- **Row-column interleaver is mildly recommended.** It provides similar performances as the "golden angle" with a slight improvement in the permutation representation and memory access for the implementation.
- **The adoption of an inter-frame interleaver is not recommended**. It has impact of memory and latency of the system and provides performance improvements in scenarios corresponding to large baud rates and/or low SNR oscillation frequencies, which are not considered relevant at the moment.

References

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[RD-5] RESCUe. ESA Study by A.Ardito-ARPSOFT, F. Chiaraluce-CNIT and L. Simone-TAS-I. REliable TT&C during Superior Solar Conjunctions. **Executive Summary**. Issue 1.1 dated 08.06.2017