

## **F8 ORBIT DETERMINATION (OD) PARAMETERS**

Satellite orbit determination (OD) estimates the position and velocity of an orbiting object from discrete observations. The set of observations includes external measurements from terrestrial or space-based sensors and measurements from instruments on the satellite itself. Satellite orbit propagation estimates the future state of motion of a satellite whose orbit has been determined from past observations. Though a satellite's motion is described by a set of ideal equations of motion representing physical hypotheses, the observations used in OD are subject to systematic and random uncertainties. Therefore, OD and propagation are probabilistic and can only approximately describe the satellite's motion. The degree of approximation that can be tolerated depends on the intended use of the orbital information.

Satellite owners/operators employ different techniques to determine orbits from active and passive observations, such that the same data inputs lead to different predictions when they are used in different models. Satellite owners/operators often accept orbit descriptions developed using physical models that others employ. Differences in orbit predictions caused using different physical models and numerical techniques can be significant.

A spacecraft is influenced by a variety of external forces, including terrestrial gravity, atmospheric drag, multibody gravitation, solar radiation pressure, tides, and spacecraft thrusters. Selection of forces for modelling depends on the accuracy and precision required from the OD process and the amount of available data. The complex modelling of these forces results in a highly nonlinear set of dynamical equations. Many physical and computational uncertainties limit the accuracy and precision of the spacecraft state that can be determined. Similarly, the observational data are inherently nonlinear with respect to the state of motion of the spacecraft and some influences might not have been included in models of the observation of the state of motion.

Satellite OD and propagation are stochastic estimation problems because observations are inherently noisy and uncertain and because not all phenomena that influence satellite motion are clearly discernible. Estimation is the process of extracting a desired time-varying signal from statistically noisy observations accumulated over time. Estimation encompasses data smoothing, which is statistical inference from past observations; filtering, which infers the signal from past observations and current observations; and prediction or propagation, which employs past and current observations to infer the future of the signal.

### **F8.1 INITIAL OD (IOD)**

Initial OD (IOD) methods input tracking measurements with tracking platform locations, and output spacecraft position and velocity estimates. No a priori orbit estimate is required. Associated solution error magnitudes can be very large. IOD methods are sometimes nonlinear methods and are often trivial to implement. Measurement editing is typically not performed during IOD calculations because there are insufficient observations. Operationally, the OD process is frequently begun, or restarted, with IOD. IOD methods were derived by various authors: LaPlace, Poincaré, Gauss, Lagrange, Lambert, Gibbs, Herrick, Williams, Stumpp,

Lancaster, Blanchard, Gooding, and Smith. Restarting techniques are most easily accomplished by using a solution from another technique.

## **F8.2 METHODS FOR SUBSEQUENT OD**

### **F8.2.1 LEAST SQUARES DIFFERENTIAL CORRECTION**

Least squares (LS) methods input tracking measurements with tracking platform locations and an a priori orbit estimate and output a refined orbit estimate. Associated solution error magnitudes are small when compared to IOD outputs. LS methods consist of an iterative sequence of corrections where sequence convergence is defined as a function of tracking measurement residual root mean square (RMS). Each correction is characterized by a minimization of the sum of squares of tracking measurement residuals. The LS method was derived first by Gauss in 1795 and then independently by Legendre.

### **F8.2.2 SEQUENTIAL PROCESSING**

Sequential processing (SP) methods are distinguished from LS processing methods in that batches of data are considered sequentially, collecting a set of observations over a specified time interval and batch-processing one interval after the next. SP can be thought of as a moving time window whose contents are captured and processed at intervals, independent of previously processed batches of data. The analysis does not include process noise inputs and calculations. It is in no way equivalent to filter processing, in which each new observation is added to past observations, improving estimates in a rigorous, traceable manner.

### **F8.2.3 FILTER PROCESSING**

Filter methods output refined state estimates sequentially at each observation time. Filter methods are forward-time recursive sequential methods consisting of a repeating pattern of time updates of the state of motion estimate and measurement updates of the state of motion estimate. The filter time update propagates the state estimate forward, and the filter measurement update incorporates the next measurement. The recursive pattern includes an important interval of filter initialization. Filter-smoother methods are backward-time recursive sequential methods consisting of a repeating pattern of state estimate refinement using filter outputs and backwards transition. Time transitions for both filter and smoother are dominated most significantly by numerical orbit propagators. The search for sequential processing was begun by Wiener, Kalman, Bucy, and others.

## F8.3 REQUIRED INFORMATION FOR ORBIT DETERMINATION

### F8.3.1 OBSERVATIONS

When observation data are communicated for collaborative or independent determination of satellite orbits, it is important to convey the observation types upon which that information is based. Ground-based, airborne, and space-based sensor observations are routinely used in orbit determination. These are conveyed in the CCSDS navigation family of messages using the Tracking Data Message (TDM) [Reference 9]. The following table describes some of the various observation types and sources.

#### Space surveillance observation product description

| Content   | Source  |
|---|---|
| two angles and slant range  | Radars  |
| two angles  | Baker-Nunn cameras, telescopes, binoculars, visual sightings    |
| Azimuth   | Direction finders   |
| Time of closest approach  | Radars, radio receivers [for transmitting (Doppler) satellites] |
| Range, angles, and rates  | Radars  |
| Pseudorange and carrier phase, as well as single, double, and triple differences of these basic measurement types | GPS or onboard inertial sensors                                 |
| Direction cosines   | Interferometric radars  |

### F8.3.2 OBSERVATION LOCATION INFORMATION

When data are communicated for collaborative or independent determination of satellite orbits, the following information about the observation location and measuring devices is important:

- facility location latitude, longitude, altitude, and the reference from which such are measured, (e.g., WGS-84);
- tracking station identification (ID);
- elevation cutoff;
- measurement biases;
- transponder delay for downlinked information

### F8.3.3 SATELLITE INFORMATION

When performing OD using active transponder ranging, the transponder delay must be provided.

### F8.3.4 ESTIMATION PARAMETERS AND CONTROL

When data are communicated for collaborative or independent determination of satellite orbits, the following information about estimation parameters and control are necessary, as described in Section 6.2.9:

- estimation parameters;
- global force model controls;
- integration controls;
- database controls;
- observation uncertainties.

### F8.3.5 FORCE MODEL SETTINGS

Spacecraft are affected by conservative and non-conservative forces. Non-conservative phenomena dissipate spacecraft energy, for example by doing work on and heating the atmosphere, as described in Section 6.2.8.

#### F8.3.5.1 GRAVITY

Central body gravitational fields are typically described using terms of a Jacobi polynomial expansion of finite order and degree. Jacobi polynomials are a complete, orthonormal set over the unit sphere. There are two angular degrees of freedom, equivalent to latitude and longitude. Any analytic function within that space can be represented by a weighted doubly infinite series of Jacobi polynomials.

**Two-body motion or Keplerian motion** considers only the point-mass gravity of the attracting body. Both the spacecraft and the central body are considered point masses, with all mass concentrated at their centres of mass. This is the lowest- order zonal harmonic approximation.

A **J2 zonal perturbation** (first order) accounts for secular (constant rate over time) variations in the orbit elements due to central body oblateness, mainly nodal precession and rotation of the semi-major axis of orbit elements that are otherwise those of unperturbed, Newtonian orbits. J2 is a zonal harmonic coefficient in an infinite Jacobi polynomial series representation of the central body's gravity field. The even zonal harmonic coefficients of the gravity field are the only coefficients that result in secular changes in satellite orbital elements. The J2 propagator includes only the dominant first-order secular effects.

For **generalized spherical harmonics**, it is impractical to determine the weights (coefficients) for a mathematically complete Jacobi polynomial series representation; therefore, the series is truncated at meaningful (in terms of precision of the representation of the gravity field) order (latitudinal) and degree (longitudinal). Where practical, it is recommended to use the full degree and order of the determined spherical harmonics field, as further truncation leads to introduction of non-conservative forces and deviation from the intended fidelity of the gravitational model.

If the order and degree are equal, the truncation is "square." Since gravitational and other perturbations are not necessarily symmetrical in latitude and longitude, the best approximation for a given application is not necessarily square. The GRAVITY\_MODEL keyword in Section 6.2.8 can specify (independently) the degree and order that are used.

Static elements of the gravity field are the gravitation of the fixed portions of the distribution of the Earth's mass. The static gravity field is not uniform. Dynamic elements of the gravity are caused by the fluid elements of the Earth's core and by variations in the distribution of water. There are solid and ocean tides. The OCEAN\_TIDES\_MODEL and SOLID\_TIDES\_MODEL of Section 6.2.8 can be used to specify these settings.

**Multibody gravitation:** Certain phenomena, such as libration points, only exist with more than two gravitationally interacting bodies. It is therefore important to describe information about third-body or multiple-body gravitational interactions if such are considered. The N\_BODY\_PERTURBATIONS keyword in Section 6.2.8 is used to specify which bodies were modelled.

### **F8.3.5.2 ATMOSPHERIC RESISTANCE (“DRAG”)**

Gas-dynamic resistance can be a significant dissipative force in low Earth orbits (LEOs). It is usually sufficient to represent it as aerodynamic drag, the product of dynamic pressure, aggregated drag coefficient, and cross-sectional area.

#### **F8.3.5.2.1 DRAG COEFFICIENT**

Drag coefficient depends upon satellite geometry, orientation, and gas-dynamic regime described by Knudsen number (ratio of object characteristic dimension to gas mean free path) and Mach number (ratio of object speed to acoustic propagation speed). When describing how atmospheric resistance is represented, data providers provide the value of drag coefficient employed using the keyword in Section 6.2.8.

#### **F8.3.5.2.2 ATMOSPHERIC DENSITY MODEL**

Density within the Earth's atmosphere varies temporally and spatially. Those variations are important in LEO. Some acceptable and most-often used atmospheric density models are as follows (although many may be utilized, as specified by the ATMOSPHERIC\_MODEL keyword in Section 6.2.8):

- 1976 Standard Harris-Priester;
- Jacchia 1970 and 1971;
- Jacchia-Roberts;
- MSIS (Several Versions and extensions).

These models typically require measurable input parameters that are “proxies” for the variation of atmospheric parameters. These include solar flux/geomagnetic particle flux which can be inferred from the meteorological observables as set by the keyword SW\_DATA\_SOURCE or FIXED\_YYYYY of Section 6.2.8:

- daily F10.7,
- average F10.7, and
- geomagnetic index.

### **F8.3.5.3 RADIATION PRESSURE**

Momentum transfer from photons to satellites can be an important force for HEOs. Radiation pressure depends on the area and surface characteristics of the satellite and the nature of the incident radiative fluxes. The Sun is the predominant direct source of electromagnetic radiation, but the Earth and the Moon also emit and reflect electromagnetic radiation. The keywords provided in Section 6.2.8 (SHADOW\_BODIES, SRP\_MODEL, ALBEDO\_MODEL, ALBEDO\_GRID\_SIZE) allow the user to specify radiation pressure settings used in the OD and orbit propagation regarding:

- solar radiation pressure coefficient;
- area-to-mass ratio;
- satellite bidirectional reflectance function (BDRF) or equivalent;
- shadow and shape factor models;
- eclipse models (cylindrical, dual-cone);
- Albedo and intensity at the satellite.

### **F8.3.6 ORBIT PROPAGATION**

Orbit propagation or prediction has evolved synchronously with advances in computational capability. Initially, force models were greatly simplified, and most important non-gravitational forces were approximated analytically. These generally linearized approaches were valid only over short intervals or for small variations from two-body Keplerian motion.

Even when more precise numerical integration became feasible, execution times were often too long, and computation was too expensive to employ numerical integration on a regular basis. Several semi-analytical techniques emerged that reduced numerical complexity (with some compromise to precision) by providing formulae from which significant elements of the propagation workflow could be extracted.

Purely numerical techniques are not used frequently. These suffer only the physical approximations made in describing important phenomena and numerical phenomena common to all discrete computations. Analytical, numerical, and semi-analytical orbit propagation techniques are distinguished. Semi-analytical and analytical approaches are specific “propagators”. This subclause applies to numerically derived orbit predictions.

### **F8.3.7 ORBIT ELEMENTS**

Orbit elements are the sets of parameters that emerge from the smoothing, filtering, or predictive estimation schemes. Six independent quantities and orbit elements describe the orbit of a satellite. A seventh variable designates the satellite location at a specific time of interest (epoch). There are many different sets of orbit elements (see orbit element set type; selected per ANNEX B, Section B7). Each is best suited for a particular application, such as aiming antennas, ease of manipulation in various coordinate systems, or estimating orbits from different types of measurements.

The traditionally used set of orbital elements is called the set of Keplerian elements; Keplerian elements parameters can be encoded as text in several formats. In semi-analytical propagation, mean orbit elements are often used, the most common of them is as conveyed in the NASA/NORAD “two-line elements” (TLE) format, originally designed for use with 80-column punched cards (but still in use because it is the most common format).