### attitude representations

Several attitude representations are available, and the particular representation is generally chosen to suit the attitude stabilization mode of the spacecraft. Examples of stabilization modes include:

* single axis (spinning);
* three axis;
* gravity gradient;
* uncontrolled.

Because of this wide domain of configuration it is convenient to use a single representation to describe the status of the attitude (see reference [17] for a survey of attitude representations). This mathematical representation of a rigid body’s attitude is called a ‘quaternion’. As it is non-ambiguous and singularity free, it is the most convenient for attitude kinematics, and can be used for every attitude stabilization mode. Quaternions unambiguously define the attitude; however, because in some cases where attitude cannot be completely determined, or complete attitude determination is not required, attitude may not unambiguously define a quaternion.

The attitude elements needed are as follows:

* given time;
* quaternion at the given time;
* description of body frame;
* description of reference frame.

NOTE – Often the reference frame is an inertial reference frame (J2000, for example). In the following, the transformations will be written generically as transforming from a frame A to a frame B.

The attitude of the body frame with respect to the reference frame is represented by a unique rotation around a vector **e**, which is invariant in both frames, with an angular amplitude Φ. The vector **e** is oriented in such a way that makes Φ positive directly around the **e** vector in the movement from reference frame to body frame.

At this rotation is associated a unit quaternion Q. The scalar component of this 4-vector, cos(Φ/2) ≡ QC, is conventionally written as either the first or last component. Care must be taken to ensure that the same convention is used by exchange participants. In the following description the convention placing the scalar last is used, and is consistent with the definition in the SANA registry.

The attitude quaternion is defined by a 4-dimension vector with:

Q1 = e1\*sin(Φ/2);

Q2 = e2\*sin(Φ/2);

Q3 = e3\*sin(Φ/2);

QC = cos(Φ/2).

Where Φ is the Euler rotation angle between the reference frame and the body frame and e1, e2, and e3 are the components of the Euler unit rotation vector **e** in the body axis (or in the reference frame) with the relation

Q12+ Q22 + Q32+ QC2 = 1

Also defined is the conjugate quaternion

With

the components of a vector in the reference frame, with = (1,2,3)

the components of a vector in the body frame, with = (1,2,3)

and are linked by

and

(These products are defined by the quaternion algebra.)

The above transformation can equivalently be represented using a direction cosine or rotation matrix, *M****BA,*** to transform from frame A to frame B, where MBA is a function of the quaternion components.

= MBA \*

The following formulae give the relations for the associated quaternion:

Q1 = (M23- M32) / (4 \* QC)

Q2 = (M31 - M13) / 4 \* QC)

Q3 = (M12 - M21) / (4 \* QC)

QC2 = (M11 + M22 + M33 + 1)1/2 / 2

The MBA matrix can be used to elaborate a set of attitude angles like Euler’s (Roll, Pitch, Yaw) giving the rotation angles around X=1=roll, Y=2=pitch, Z=3=yaw axes. The rotation order must be defined to have a set of values consistent with the desired rotation.

For example, if the rotation order is Φ around axis 3, followed by Θ around axis 2, followed by Ψ around axis 1, the Euler angles Φ, Θ, and Ψ can be obtained by the following relations:

NOTE – The angles Φ and Ψ are undefined if cos(Θ) = 0. Several solutions are possible depending on the quadrants in which the inverse trigonometric function solutions are taken.

The transformation matrices above, while assuming a particular order of vector (Q1, Q2, Q3) and scalar (QC) portions of the quaternion, are invariant as it is an expression to transform a vector between the two frames of the quaternion. If one were to attempt quaternion multiplication, then order of the quaternion vector is of prime importance.

Another method of expressing 3-axis attitude is given in terms of roll, pitch, and yaw (R, P, Y) coordinates. However, the definition of roll, pitch, and yaw axes vary from mission to mission and often even change within a mission (reference [16]). In determining rotations or rotation rates about these axes, care must be taken to define the axes, which are often misunderstood. Using quaternions avoids the need to use R, P, Y attitude representation and its resulting complexity.

Spin-stabilized spacecraft rotate about a known body-frame spin-axis. Their attitude is often represented as the orientation of the spin-axis with respect to an external reference frame, a phase angle of rotation about this axis, and/or a spin rate about this axis. Generally, the phase of rotation about the spin axis is neither determined nor needed. In this case the spacecraft attitude is sufficiently determined by only two parameters, and a quaternion is not unambiguously specified by the conventionally determined attitude. In this case the most common attitude representation is the right ascension and declination of the spin-axis in the reference frame.