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COORDINATE SYSTEMS FOR SPACE AND GEOPHYSICAL APPLICATIONS

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13. ABSTRACT (Maximum 200 words) Many coordinate systems used at Phillips Laboratory, Hanscom AFB for space and geophysical applications are described. This document describes astronomical, orbital, and geomagnetic systems, and special emphasis is given to magnetospheric coordinates in which radiation belt particle investigations may be made. An extensive list of terms and abbreviations is included. These coordinate systems are identified by their origins and the primary and secondary reference axes, which allows them to be systematically compared, categorized and diagrammed. Angle coordinate systems for defining attitude are intimately related, and are also covered. Transformation matrices are provided for several systems, for conversion from one coordinate system into another.

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TABLE OF CONTENTS

TABLE OF CONTENTS (cont'd)

LIST OF FIGURES

LIST OF FIGURES (cont'd)

LIST OF TABLES

1. INTRODUCTION

Phillips Laboratory, Hanscom AFB uses many different coordinate systems in the analysis, processing and display of data acquired in aerospace experiments. In particular, emphasis is placed on parameters that relate to astronomical or spacecraft ephemerides and to geomagnetic fields. The requirements of the CRRES satellite experiment accelerated the need for identifying magnetospheric coordinate systems that were best suited to define electron and ion distributions in the radiation belts. Past satellite and aircraft experiments, as well as current Orbiter applications [*CIRRIS*, 1986] provided additional sources, and a compilation [*Cottrell and McInerney, 1985*] was prepared, describing many of the coordinate systems that have been or could be used. As space research and intricate manned missions progress, angular rotation angles that define instrument look directions as well as spacecraft orientations tend to be intimately related. These angle or attitude coordinates are briefly discussed, although detailed coverage is outside the scope of this effort.

The present document is the result of considerable editing of the various sections: the descriptions, glossary and bibliography are extended; the basic categorization method was used to eliminate extraneous coordinate systems; the diagram presentations are improved, coordinate transformations are defined more consistent with mathematical notation; and epochs and coefficients have been brought up to date 1990.0. Definitions of terms and symbols that are prevalent and that are used subsequently are presented in Chapter 2.

With so many coordinate systems, confusion often arises as to which coordinate system names go with which coordinate system descriptions. In an attempt to improve clarification, Table 3-1 in Chapter 3 relates each coordinate system name with its fundamental coordinate system reference frame, *viz.* origin, primary and secondary axis. Table 3-2 lists many of the aerospace applications of interest, and identifies the coordinate systems which would be pertinent. These tables afford a general overview of all the systems. Following the tables, specific descriptions of the systems are included, along with diagrams.

Chapter 4 describes coordinate system transformations, particularly those from the Earth Centered Inertial system, which is frequently used as the fundamental system associated with raw experimental data, to several of the coordinate systems described in Chapter 3. It is shown that other coordinate transformations can always be accomplished through intermediate transformations into ECI coordinates. Subsequently, a separate section discusses some angular rotation transformations that are pertinent to attitude determination.

Chapter 5 provides reference material which discusses the basis of the Geomagnetic Dipole models. The centered dipole model is readily derived from the first order representation of Earth's magnetic field. For space particles up to geosynchronous altitude, the eccentric dipole may provide a superior magnetic reference frame. The dip pole model, of interest only near Earth's surface, is presented for completeness.

2. TERMINOLOGY AND ABBREVIATIONS

This chapter is included in order to define terms and abbreviations that are used in chapter 3. In many cases the generality of a definition is restricted to application to the Earth only, as opposed to other celestial bodies.

a - Abbreviation for semi-major axis of orbit.

Aberrated Coordinates - Refers to coordinate systems such as Solar Wind and Solar Wind Magnetospheric in which the primary axis points to a position that deviates from the expected position (e.g., points to solar wind rather than Sun).

Anomaly, Eccentric - Angle measured in the Orbital Plane, from Perigee to the vector from the center of the major axis to the fictitious point defined by projecting the spacecraft onto the auxiliary circle that circumscribes the ellipse of motion, in a direction perpendicular to the major axis. The Eccentric Anomaly (EA) is related to the Mean Anomaly (MA) by Kepler's Equation:

$$MA = EA - e \sin (EA)$$

Anomaly, Mean - A fictitious angle measured from Perigee in the Orbital Plane, uniformly increasing with time, positive in the direction of motion.

Anomaly, True - [*CIRRIS*, 1986] The geocentric angular displacement of a space vehicle measured from Perigee in the Orbital Plane, and positive in the direction of travel in the orbit.

AP - Abbreviation for Argument of Perigee. (See Perigee, Argument of).

Apogee - [*Marks*, 1969] The point in a geocentric orbit which is most distant from the Earth.

ATD - Aries-True-of-Date (inertial coordinate system).

ATS-1 - Applications Technology Satellite (quasi-inertial coordinate system).

Axis, Dipole - [*Marks*, 1969] The axis of a magnetic dipole assumed to be located at the center of the Earth and that is a first order approximation to the actual magnetic field of the Earth.

Axis, Rotational - Refers to Earth's rotational axis.

AZ - Abbreviation for Azimuth.

Azimuth - [*Marks*, 1969] Angle measured in the horizontal plane, at the observer's location (the origin), from a horizontal reference vector (usually north), to the projection of the Radius Vector to the point of interest onto the horizontal plane (positive clockwise).

B - Magnitude of the magnetic field vector B.

$\overline{B}$ - Abbreviation for magnetic field vector.

BA - Abbreviation for Body Axis (Orbiter body coordinate system).

Celestial Sphere - [*Marks*, 1969] Hypothetical celestial background concentric with the observer or other reference point of interest, such as the center of the Earth [*Riordan*, 1966]. The celestial bodies and the paths along which they move may be projected on this infinite radius sphere using spherical trigonometry.

CGM or CGEOM - Corrected Geomagnetic (see Geomagnetic Coordinates, Corrected)

CGMLT - Corrected Geomagnetic Local Time.

D - Dipole (geomagnetic coordinate system).

DEC - Abbreviation for Declination and Magnetic Declination.

Declination - [*CIRRIS*, 1986] Angle between the Radius Vector and the Equatorial Plane.

Declination, Magnetic - [*McInerney, et. al.*, 1973] Angle between the horizontal component of the magnetic field and geographic north (positive towards east). Effectively the azimuth.

Dip, Magnetic - [*Riordan*, 1966] Angle between the magnetic field vector and the local horizontal plane (positive downwards). Same as Inclination, Magnetic.

Dipole, Magnetic - (See Axis, Dipole).

DM - Dipole Meridian (geomagnetic coordinate system).

e - Abbreviation for eccentricity of the orbit.

EA - Abbreviation for Eccentric Anomaly.

ECI - Earth Centered Inertial (inertial coordinate system).

ECL - Ecliptic (quasi-inertial coordinate system).

Ecliptic - [*Riordan*, 1966] Plane defined by the apparent annual path of the Sun around the Earth.

EL - Abbreviation for Elevation.

Elevation - [*Riordan*, 1966] Angle of a point of interest above local horizontal reference plane.

Ellipsoid, Reference - Fischer's ellipsoid characterizes the Earth's surface. The Astronomical Almanac adopts the IAU (1976) System of Astronomical Constants, which specifies the semimajor axis (equatorial) to be 6378.140 km and the flattening to be 1/298.257, giving 6356.755 km for the semiminor axis (polar). For the CRRES satellite ephemeris, R_E is defined as 6378.135 km, and the flattening is defined as 1/298.26.

Epoch - [*Riordan*, 1966] An arbitrary instant in time for which the position of points planes, or directions in space are valid.

EP - Equatorial Plane.

Epoch 1950 - [*Riordan*, 1966] This Epoch is the beginning of Besselian year 1950 and is equivalent to the Julian ephemeris date 2433282.423357 (the number of elapsed days plus fraction since Noon on January 1, 4713 B. C.

Equator - The primary great circle that is perpendicular to the Earth's Rotational Axis and is 90° from the Geographic Poles.

Equator, Celestial - [*Riordan*, 1966] Intersection of the Earth's Equatorial Plane with the Celestial Sphere.

Equator, Epoch - Refers to Earth's Equator for a specific Epoch.

Equator, Geomagnetic - [*Marks*, 1969] Terrestrial great circle everywhere 90° from the Geomagnetic Poles. Usually defined by the centered dipole field.

Equator, Magnetic - [*Marks*, 1969] Line on Earth's surface connecting all points of zero Magnetic Dip. Found from actual main geomagnetic field.

Equator, Mean or Mean-of-Date - [*Riordan*, 1966] Position of the Equator at a specified time including the effects of Precession, but excluding the effects of Nutation.

Equator, True or True-of-Date - [*Riordan*, 1966] Position of the Equator at a specified time including the effects of both Precession and Nutation.

Equinox, Epoch - Refers to a Vernal Equinox for a specific epoch.

Equinox, Mean or Mean-of-Date - [*Riordan*, 1966] Position of the Vernal Equinox at a specified time excluding the effect of Nutation.

Equinox, True or True-of-Date - [*Riordan*, 1966] Position of the Vernal Equinox at a specified time including the effects of both Precession and Nutation.

Equinox, Vernal - The point on the Celestial Sphere at the intersection of the Celestial Equator and the Ecliptic, where the Sun crosses the Equator from south to north in its apparent annual motion along the Ecliptic [*Riordan*, 1966].

Euler Angles - Three orthogonal rotational angles, e.g. yaw, pitch and roll, that suffice to turn an object from any initial orientation to any other arbitrary orientation. The rotation sequence must be specified (or implied), since the same rotations in say, pitch-yaw-roll sequence, do not result in the identical final orientation.

Euler Axis and Angle - A fixed axis and a finite rotation angle, constituting a quaternion, suffice to turn an object from any orientation to any other.

First Point of Aries - or the Vernal Equinox.

FK5/J2000 - Vernal Equinox at Epoch 2000.0 (Inertial coordinate system).

Flight Path Angle - Angle between the reference plane and the velocity vector in a velocity system (positive upwards).

FPA - Abbreviation for flight path angle.

GCI - Geocentric Celestial Inertial (coordinate system).

GEI - Geocentric Equatorial Inertial (coordinate system).

GEOC - Geocentric (Inertial or Earth-fixed coordinate system).

GEOD - Geodetic latitude, longitude, and altitude for oblate Earth (Earth-fixed coordinate system).

Geodetic Altitude - [*CIRRIS*, 1986] Distance from the point of interest to the Reference Ellipsoid, measured along the local vertical (positive for points outside the Reference Ellipsoid).

GEOG - Geographic, usuall Geodetic.

GEOM - Geomagnetic (Earth-fixed coordinate system).

Geomagnetic Coordinates - Latitude, longitude, and radial distance, typically in a geocentric, geomagnetic dipole reference frame.

Geomagnetic Coordinates, Corrected - Latitude and longitude in the asymmetric geomagnetic field, ususally applied at low altitudes [*Gustafsson*, 1970].

GSE - Geocentric Solar Ecliptic (quasi-inertial coordinate system).

GSEQ - Geocentric Solar Equatorial (quasi-inertial coordinate system).

GSM - Geocentric Solar Magnetospheric (geomagnetic coordinate system).

h - Symbol for Geodetic Altitude.

Hour Angle - Angle measured along the Equatorial Plane from the Local celestial Meridian to the Meridian containing the Vernal Equinox (positive towards west).

Hz - Abbreviation for horizontal component of magnetic field.

INC - Abbreviation for Inclination and Magnetic Inclination.

Inclination - [*Marks*, 1969] Angle between the satellite Orbital Plane and the Equatorial Plane. The value ranges between 0° and 180°.

Inclination of Orbital Plane - [*CIRRIS*, 1986] Angle between the north polar axis and the orbital angular momentum vector.

Inclination, Magnetic (or Magnetic Dip Angle) - [*McInerney, et. al.*, 1973] Angle measured from the horizontal component (Hz) of the magnetic field to the total vector of the magnetic field (positive downwards). At the magnetic poles, the magnetic dip is 90°.

L - Abbreviation for L-shell.

LA - Look Angle (Orbiter body coordinate system).

LAT - Abbreviation for latitude.

Latitude, Celestial - [*Riordan*, 1966; *Escobal*, 1965] Often refers to Ecliptic Latitude. Use of this term should be avoided because of ambiguity.

Latitude, Ecliptic - Angle between ecliptic plane of epoch and Radius Vector of interest. Positive in the direction of the North Ecliptic Pole.

Latitude, Geocentric - [*Riordan*, 1966] Angle measured at the center of Earth between the Equatorial Plane and the Radius Vector to the point of interest.

Latitude, Geodetic - [*Riordan*, 1966] Angle between Equatorial Plane and the normal to the Reference Ellipsoid at the point of interest.

Latitude, Geographic - [*Marks*, 1969] Same as Geodetic Latitude.

Latitude, Geomagnetic - [*Riordan*, 1966] Angle measured along a Geomagnetic Meridian north or south of the Geomagnetic Equator.

Latitude, Invariant - [*McInerney, et. al.*, 1973] LAT= $\pm \cos^{-1}(1/L)$ where L is L- shell.

Latitude, Magnetic - [*Marks*, 1969] Angle measured to the Magnetic Field Vector from the projection of the Magnetic Field Vector onto the horizontal plane.

Latitude, Magnetic Dip - (Same as Magnetic Latitude).

LO - Local Orbital (Orbiter coordinate system).

Local Time, Geomagnetic - [*McInerney, et. al.*, 1973] Defined as one-fifteenth of (geomagnetic longitude of a satellite, minus the geomagnetic longitude of the Sun, plus 180°) (in hours).

Local Time, Magnetic - Defined as one-fifteenth of (magnetic longitude of a satellite, minus the magnetic longitude of the Sun, plus 180°) (in hours).

Local Vertical, Geodetic - Refers to the perpendicular to the Reference Ellipsoid of the Earth that passes through the point of interest.

LON - Abbreviation for longitude.

Longitude - [*Marks*, 1969] Angle between a reference plane through the polar axis (e.g., Greenwich Meridian) and a secondary plane through that axis (e.g., Local Meridian) (positive towards east).

Longitude, Ascending Node - [*Riordan*, 1966] Angle measured eastward from the Vernal Equinox in the Equatorial Plane to the Ascending Node.

Longitude, Celestial - [*Riordan*, 1966; *Escobal*, 1965] Often refers to Ecliptic Longitude. Use of this term should be avoided because of ambiguity.

Longitude, Descending Node - Angle measured eastward from the Vernal Equinox in the Equatorial Plane to the Descending Node.

Longitude, Ecliptic - [*Riordan*, 1966; *Escobal*, 1965] Angle between projection of the Radius Vector of interest onto the ecliptic plane of epoch and the Vernal Equinox. Measured eastward from the Vernal Equinox.

Longitude, Geomagnetic - [*Riordan*, 1966] Angle measured eastward along the Geomagnetic Equator from the Meridian half-plane containing the Magnetic Poles and the Geographic South Pole.

L-Shell - A parameter which characterizes the magnetic field line passing through the point of interest and/or trapped particles mirroring at said point. There are several definitions in use, which are equivalent for a dipole field:

LVLH - Local Vertical Local Horizontal (Orbiter coordinate system).

L_m - or L-Shell - the definition given by McIlwain [1961] - the equatorial radius, in Earth radii, of a dipole field line passing through a point with the same longitudinal invariant integral I and field strength B in a dipole field as the point of interest in the actual field. The longitudinal invariant integral I is the familiar integral appearing in the second adiabatic invariant:

$$I = \int \left(\frac{1-B(s)}{B_m} \right)^{1/2} ds,$$

where the integration is along the field line between the point of interest and its conjugate point, B(s) is the magnetic field strength at a point along the field line, and B_m is the field strength at the point of interest (same as B above). This definition cannot be formulated analytically. McIlwain [1961,1966] and Hilton [1971] have given analytic approximations for L_m as a function of I and B. Stern [1968] has given an approximation for L_m for perturbed dipole fields, good to first order in the perturbation, which circumvents the need to compute I. However, this approximation is valid only for fields expressible as the gradient of a potential, i. e., for fields in current-free environments. The perturbation approach could possibly be generalized for application to more arbitrary fields.

L_e - the equatorial radius of the actual field line passing through the point of interest [*Stone*, 1963].

L_0 - the McIlwain L value at the equatorial crossing point of the field line passing through the point of interest [*Stone*, 1963].

L_d - the equatorial radius of the dipole field line which the actual field line through the point of interest approaches in the high latitude limit [*Schulz and Lanzerotti*, 1974).

M - Abbreviation for a 3 x 3 transformation matrix. M^I is inverse of M.

MA - Abbreviation for Mean Anomaly.

MAG - Magnetic, used loosely for symmetric dipole, eccentric dipole, or dip pole coordinate systems.

Magnetic Dip Angle - Same as Magnetic Inclination.

Meridian - [*Marks*, 1969] A great circle that passes through the normals (e. g., the Earth's Rotational Poles) to a reference plane (e.g., the Equatorial Plane). Normally, Meridian refers to the great semi-circle which passes through a given place.

Meridian, Greenwich - [*Marks*, 1969] The Meridian that passes through Greenwich, England and the Geographic Poles.

Meridian, Local - [*Marks*, 1969] Meridian passing through a local point of interest.

Meridian, Local, Ellipsoidal - The Meridian that passes through the Geographic Poles and a local point of interest.

Meridian, Geomagnetic - A Meridian that passes through the Geomagnetic Poles.

Meridian, Local Noon - The Meridian that contains the Radius Vector of the Sun.

Meridian, Magnetic - A Meridian that passes through the Magnetic Poles.

MFD - Magnetic Field (local North-East-Vertical coordinate system).

MLT - Magnetic Local Time.

MP - Abbreviation for Meridian Plane.

Meridian, Prime - Same as Greenwich Meridian.

Meridian, Prime Geomagnetic - The Meridian that passes through the Geomagnetic Poles and the Geographic south pole. Geomagnetic longitude is measured eastward from this Meridian.

M50 - Vernal Equinox at Epoch 1950.0 (Inertial coordinate system).

n - Abbreviation for mean motion of a satellite. Usually in revolutions/day or radians/sec.

Node, Ascending - [*Riordan*, 1966] Point of intersection of the Orbital Plane of a space vehicle and the Equatorial Plane where the object crosses from south to north.

Node, Descending - Point of intersection of the Orbital Plane of a space vehicle and the Equatorial Plane where the object crosses from north to south.

Nutation - [*Riordan*, 1966; *Marks*, 1969] Oscillation of Earth's Rotational Axis about the Ecliptic Pole caused by solar-lunar-Earth interaction. Maximum displacement is about 17 arc seconds in longitude and 9 arc seconds in latitude with a period of 18.6 years [*Pratt*, 1988].

Orbiter - Refers to the Space Shuttle.

P - Abbreviation for a point of interest in any coordinate system.
or
P - Plasma (Magnetic field oriented wave propagation coordinate system).

Pericenter - [*Riordan*, 1966] Point of an orbit which is nearest the center of mass; in the case of Earth, Perigee is the Pericenter.

Perigee - [*Marks*, 1969] The point in a geocentric orbit which is closest to the Earth's center.

Perigee, Argument of - [*CIRRIS*, 1986] Angle measured in the Orbital Plane between the Ascending Node and Perigee, positive in the direction of travel in the orbit. In the case of zero eccentricity, Perigee is defined to be at the Ascending Node. Value ranges between 0° and 360°.

Plane, Celestial - Plane that contains the Celestial Equator.

Plane, Ecliptic - [*Escobal*, 1965] Plane defined by the apparent motion of the Sun during the year. Since this plane changes slightly as the result of lunar/solar precession, nutation and planetary perturbations, a standard ecliptic plane is often used (M50, J2000.0, mean-of-date, etc.).

Plane, Equatorial - [*Riordan*, 1966] Refers to Earth's Equatorial Plane. The plane that is perpendicular to the Earth's Rotational Axis and passes through the center of the Earth.

Plane, Geomagnetic Equatorial - Plane that is perpendicular to the geomagnetic axis (magnetic dipole axis) and passes through the center of the Earth.

Plane, Magnetic Equatorial - Plane that contains the Magnetic Equator.

Plane, Geomagnetic Meridian Half- - [*Riordan*, 1966] The great semi-circle that contains the Magnetic Dipole Axis and the South Geographic Pole.

Plane, Orbital - Plane defined by the Earth's orbit around the Sun or by a satellite's orbit around the Earth.

Plane, Solar Equatorial - Plane that is perpendicular to the Sun's rotational axis and passes through the Sun's center. The Sun's rotational axis points to 286.1° Right Ascension, and 63.85° Declination.

Pole, Celestial - The projection of the Earth's rotational axis on the celestial sphere. The North Celestial Pole is near the star Polaris.

Pole, Ecliptic - [*Marks*, 1969] Either of two points 90 degrees from the Ecliptic on the Celestial Sphere. The North Ecliptic Pole is located about 23° away from the North Celestial Pole.

Pole, Geographic - [*Marks*, 1969] Either of two points of intersection on the Earth's surface of a line coincident with the Earth's Rotational Axis.

Pole, Geomagnetic - [*Marks*, 1969] The intersections with the Earth's surface of a line that represents the effective axis of a simple dipole that approximates the actual magnetic field of the Earth. The Geographic Latitude and Longitude for the north geomagnetic pole is 79.186° north, 70.977° west (289.023° east), and for the south geomagnetic pole is 79.186° south, 109.023° east (IGRF85 propagated to Epoch 1990).

Pole, Magnetic - [*Marks*, 1969] Either of two points of intersection of a line with the Earth's surface where the line parallels the 90° magnetic dip of the geomagnetic field. The geographic latitude and longitude for the north magnetic pole is 77.76° north, 103.68° west, and for the south magnetic pole is 64.90° south, 139.37° east (IGRF85 at Epoch 1990). Note that these poles are not geocentrically opposed.

Pole, Rotational - Either of two points of intersection of a line along Earth's Rotational Axis with Earth's surface.

PQW - Orbital plane quasi-inertial Earth-centered coordinate system. P points towards perigee, Q is in the orbit plane advanced by 90° from P in the direction of increasing true anomaly, and W is normal to the orbit plane, completing the right-handed system [*Escobal*, 1965].

Precession - [*Riordan*, 1966] General, secular, westward motion of the Vernal Equinox along the Ecliptic which amounts to an angular change of 50.2 seconds per year or a complete revolution in about 26,000 years.

Quaternion - A four parameter entity comprised of scalar and vector components that represents any rigid body rotation, e.g. $\bar{q} = q_0 + q_1 i + q_2 j + q_3 k = (q_0, \bar{q})$. Counter to expectation, $\bar{q}$ and q_0 are not the Euler axis and rotation angle, instead these parameters are constrained by the identity $q_0^2 + q_1^2 + q_2^2 + q_3^2 = 1$.

R - Magnitude of the Radius Vector, $\bar{R}$.

$\bar{R}$ - Abbreviation for Radius Vector.

RA - Abbreviation for Right Ascension and Right Ascension of Ascending Node.

Radius Vector - The vector that originates at the center of a coordinate system and ends at a point of interest.

Right Ascension - [*CIRRIS*, 1986] Angle between projection of the Radius Vector onto the Equatorial Plane and the Vernal Equinox of epoch. Measured eastward from the Vernal Equinox.

Right Ascension of Ascending Node - [*CIRRIS*, 1986] Angle measured eastward from the Vernal Equinox of epoch in the Equatorial Plane to that intersection with the Orbital Plane where the spacecraft passes from south to north. In the case of zero Inclination, the Ascending Node is defined to be the X-axis of the reference system.

S - Sensor (Instrument-fixed coordinate system).

SE - Solar Ecliptic (quasi-inertial coordinate system).

SG - Solar Geomagnetic (coordinate system).

SM - Solar Magnetic (coordinate system).

SMC - Solar Magnetospheric (coordinate system).

SW - Solar Wind (aberrated coordinate system).

SWM - Solar Wind Magnetospheric (aberrated coordinate system).

TA - Abbreviation for True Anomaly.

TE - Topocentric Equator (Earth-fixed coordinate system).

TH - Topocentric Horizon (Earth-fixed coordinate system).

True-of-Date - Current date and time of interest with the effects of Nutation and Precession included.

UVW - Orbiter coordinate system.

V - Magnitude of the Velocity Vector, $\overline{V}$.

$\overline{V}$ - Abbreviation for Velocity Vector and abbreviation for line-of-sight vector in Look Angle coordinate system.

VDH - Geomagnetic dipole oriented local coordinate system.

Velocity Vector - Instantaneous direction and speed of a point of interest or space vehicle.

Vernal Equinox - (See Equinox, Vernal).

W - Wave (Propagation and magnetic field oriented coordinate system).

$\overline{W}$ - Abbreviation for Propagation Vector.

Zenith Angle - Angle measured from the zenith or local vertical to a point of interest. (90° minus the Elevation).

3. SPACE COORDINATE SYSTEMS

Many different coordinates systems are used to analyze, process and display data, because physical processes may be better understood if the experimental data are more ordered or if calculations are more easily performed in one coordinate system rather than another. An extensive repertoire of coordinate systems, particularly astronomical, geomagnetic and spacecraft ephemeris/attitude related systems are pertinent to the research activities at Phillips Laboratory.

Some of the individual coordinate systems have several names associated with them. For completeness, each named system is described separately and cross-referenced to other systems that have the same characteristics. Although coordinate systems are often depicted in various ways including spherical coordinates, after the origin is defined, a primary axis direction and a secondary reference orientation (or plane) is all that is needed to specify any system. Comparisons of the various rectangular coordinate systems are illustrated in Table 3-1. The primary and secondary axes are defined by specific reference positions (such as the Earth's rotational axis, the solar direction, or the vernal equinox). This approach reduces to a right-handed Cartesian system and, as may be seen by examining Table 3-1, has many advantages:

1) The fundamental origin and directions that one may need to consider are identified;

2) Possible problems of different names for the same system are resolved;

3) Coordinate conversion reduces to a matrix rotation which is implicitly defined by the fundamental directions of the pair of coordinate systems in question.

This section describes many coordinate systems that are either Earth-fixed, inertial, quasi-inertial, magnetic or in the category of "other". The range of applications is more or less self-evident, and Table 3-2 categorizes them by four major fields: astronomical, orbital, geomagnetic and local observation. Coordinates with the origin at the point of interest (PI) or center of mass (CG) are inherently suited for local observation and are the ones that are employed for attitude determination. Most, although not all, of the coordinate systems have both rectangular and spherical representations. Some of the systems have adjunctive representations in addition to rectangular and spherical.

Each system is described in terms of its origin, orientation, characteristics and applications; an illustrative figure is also included. Unless otherwise indicated, coordinate systems are true-of-date. Due to the secular variation of Earth's internal magnetic field, the geomagnetic dipole and therefore a primary reference axis for the geomagnetic coordinate systems migrates with time when referred to an Earth-fixed system. The geographic latitude and longitude of the magnetic poles have been updated here to Epoch 1990.0, by propagation of the IGRF85 model [*EOS*, 1986].

Table 3-1. Coordinate Systems (Rectangular) Comparisons

	orig	rot axis	ecl axis	vern equx	sun	grwh meri	north	geom axis	sol wind	perp rad vec	zen	perp orb pln	orb per-igee
Earth-Fixed													
GEOCENTRIC	EC	(Z)				X							
GEODETIC (1)	EC	(Z)				X							
TOPOCEN EQUAT	PI	(Z)				X*							
TOPOCEN HORIZ	PI						X*				(Z)		
Inertial/Quasi-Inertial													
EARTH CENT INERT (2)	EC	Z		(X)									
ARIES-OF-EPOCH (3)	EC	Z		(X)*									
ATS-1 (9)	EC	(Z)			X*								
ECLIPTIC (4)	EC		Z		(X)								
GEOCEN SOL EQUAT (10)	EC	Z*			(X)								
PQW	EC											Z	(X)
Magnetic													
CENTERED DIPOLE (11)	EC	X*						(Z)					
MAGNETIC (12)	MC	X*						(Z)*					
DIPOLE MERIDIAN	EC							(Z)		Y			
GEOC SOL MAGSPH (5,13)	MC/EC				(X)		Z*						
SOLAR MAGNETIC (6,14)	MC/EC				X*			(Z)					
SOL WIND MAGSPH (13)	EC						Z*		(X)				
SOLAR WIND	EC		Z						(X)				
MAGNETIC FIELD (15)	PI						(X)				-Z		
VDH	PI							(Z)		Y*			

	orig	geoc	vel vec	ang mom vec	para veh body	veh sym pln down	mag fld	prop vec	lofs	as
Other Systems										
BODY AXIS (7)	CG				(X)*	Z'				
LOCAL ORBITAL (3)	CG	(Z)*	X							
UVW	PI	(X)*		Z						
SENSOR	AS								(Z)	X*
PLASMA (16)	PI						(Z)	X*		
WAVE (17)	PI						Y*	(Z)		

Legend for Table 3-1.

X: X-AXIS; Y: Y-AXIS; Z: Z-AXIS
( ): PRIMARY DIRECTION
UNMARKED: SECONDARY DIRECTION
EC: EARTH-CENTERED
PI: POINT OF INTEREST
NEC: NEAR CENTER OF EARTH (Eccentric Dipole)
MC: MAGNETIC CENTER (can be either EC or NEC for CRRES Program)
CG: CENTER OF GRAVITY
AS: ARBITRARILY SELECTED
* : MORE PRECISE REFERENCE REQUIRED TO CLARIFY. SEE SECTIONS THAT FOLLOW.
**: OTHER REFERENCES POSSIBLE
ROT AXIS: COINCIDENT OR PARALLEL TO ROTATIONAL AXIS
ECL AXIS: COINCIDENT OR PARALLEL TO ECLIPTIC AXIS
VERN EQUX: POINTS TO VERNAL EQUINOX
SUN: POINTS TO SUN
GRWH MERI: POINTS TO GREENWICH MERIDIAN
NORTH: POINTS TO NORTH
GEOM AXIS: COINCIDENT OR PARALLEL TO GEOMAGNETIC AXIS
PERP ORB PLN: PERPENDICULAR TO ORBITAL PLANE
ORB PERIGEE: POINTS TO ORBIT PERIGEE
GEOC: POINTS TO CENTER OF EARTH
VEL VEC: ALONG VELOCITY VECTOR
ANG MOM VEC: ANGULAR MOMENTUM VECTOR
PARA VEH BODY: PARALLEL TO ORBITER STRUCTURAL BODY
VEH SYM PLN DOWN: IN ORBITER PLANE OF SYMMETRY (DOWNWARDS)
MAG FLD: PARALLEL TO MAGNETIC FIELD VECTOR
PROP VEC: PARALLEL TO WAVE PROPAGATION VECTOR
LOFS: COINCIDENT OR PARALLEL TO LINE IF SIGHT
AS: ARBITRARILY SELECTED
(1): Same as GEOGRAPHIC System.
(2): Same as GEOCENTRIC CELESTIAL INERTIAL and GEOCENTRIC EQUATORIAL INERTIAL Systems.
(3): Same as ECI System; Epcoh can be True-of-Date (ATD), Mean-of-1950 (M50), or Mean-of-2000 (FK5/J2000).
(4): Same as GEOCENTRIC SOLAR ECLIPTIC and SOLAR ECLIPTIC Systems.
(5): Same as SOLAR MAGNETOSPHERIC System.
(6): Same as SOLAR GEOMAGNETIC System.
(7): Same as LOOK ANGLE System.
(8): Same as LOCAL VERTICAL LOCAL HORIZONTAL System.
(9): X-Axis points to Local Noon Meridian and lies in Equatorial Plane.
(10): Z-Axis lies in plane containing X-axis and Sun's Rotational Axis.
(11). X Axis lies in plane containing North Dipole Axis and Rotational Axis, positive towards South Geographic Pole. Sometimes called the GEOMAGNETIC System.
(12): Z-Axis is parallel to Magnetic Dipole Axis but is magnetic centered. X-Axis is parallel to that of the Dipole coordinate system. Also sometimes called the GEOMAGNETIC System.
(13). Z-Axis lies in plane containing X-Axis and North Dipole Axis, is perpendicular to the X Axis and is positive towards North.
(14): X-Axis lies in plane containing Z-Axis and the Sun, positive towards Sun, but lies in Geomagnetic Equatorial Plane.
(15): (-Z)-Axis is perpendicular to the magnetic meridian (horiz component of B field).
(16): X-Axis lies in plane containing the Z-Axis and Propagation Vector, W.
(17). Y-Axis lies in plane containing the Z Axis and the Magnetic Field Vector, B, positive in same sense as B, Z-Axis is parallel to Wave Propagation Vector.

Table 3-2. Applications of the Various Coordinate Systems

ASTRONOMY	M50, ATD, ECI, GCI, GEI
Relates coordinate systems of rotating earth to Inertial or Quasi-Inertial systems	GEOC, GEOD, GEOG
Define the position of ground observations and transmitting and receiving stations	GEOD, GEOG
Order data controlled by the sun	GSEQ
SATELLITE ORBITS	
Convert orbital orientation to fixed-earth orientation	LO, LVLH, UVW
Display satellite trajectories	EC, SE, GSE
Satellite orbit calculations	M50, ATD, M50 (Orbital Elements) ATD (" ")
Satellite velocity calculations	GEOG (Polar Velocity) M50 (" ") ATD (" ")
Space vehicle dynamics	ECI, GCI, GEI
LOCAL OBSERVATION (ATTITUDE)	
Direction of celestial objects, ground stations, other orbiting vehicles from the Orbiter	LA
Define Orbiter-antenna radiation-distribution patterns	LA
Orient Orbiter into desired position	BA
Convert sensor data to other coordinate systems	S
Experimental measurements	TH
Local observations	TE
Navigation	MAG

GEOMAGNETIC

Order data controlled by dipole magnetic field	SG, SM, DM
Study magnetic field close to the earth	D
Define the position of magnetic observations	MAG
Used in magnetic field line tracing	MAG
Describe distortions in the magnetic field	DM
Analysis of distortions in the magnetic field close to earth	CGEOM
Analysis of magnetic data	ATS-1
Analyze components of the magnetic field	MFD
Analyze geomagnetic field data	VDH
Examine statistical relationships	GSM, SMC
Magnetopause, currents, shock boundary positions	GSM, SMC
Magnetosheath, magnetotail magnetic fields	GSM, SMC
Magnetosheath, solar wind velocities	GSM, SMC
Effects of solar wind on magnetosphere	SWM
Analyze the impact of the solar wind on hemispheric events	SW
Solar Wind velocity data	EC, SE, GSE, GSM, SMC
Study wave propagation	P (in plasma), W

3.1 EARTH-FIXED

3.1.1 GEOCENTRIC (GEOC) [*Riordan*, 1966]

ORIGIN: Center of Earth.

ORIENTATION:

X-axis: Points to Greenwich meridian and is in the equatorial plane.

Y-axis: Completes the right-handed orthogonal set $\overline{Y} = \overline{Z} \times \overline{X}$ and is in the equatorial plane.

Z axis: Coincident with the Earth's rotational axis and is positive towards north.

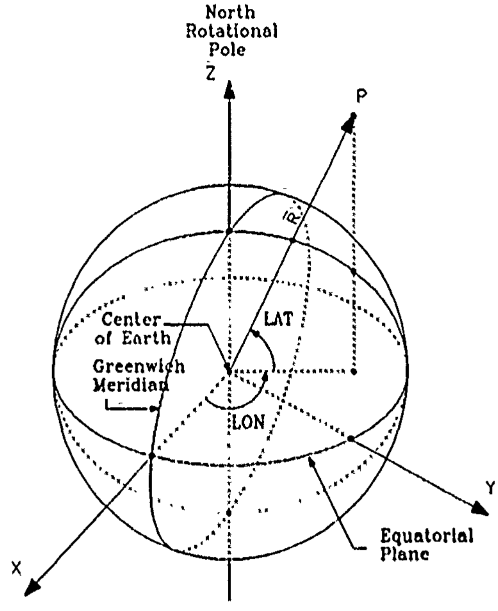

Figure 3-1. GEOC System

LAT: Angle measured at the center of the Earth between the projection of the radius vector $\overline{R}$ on the equatorial plane to the radius vector $\overline{R}$ and is positive towards north.

LON: Angle measured positive eastward along the equator from the Greenwich meridian to the local meridian of the radius vector $\overline{R}$.

R: Magnitude of the radius vector $\overline{R}$.

CHARACTERISTICS: Geocentric, Earth-fixed and rotating.

APPLICATIONS: Convenient system for relating the coordinate system of a rotating Earth to inertial and quasi-inertial systems.

3.1.2 GEODETIC (GEOD) GEOGRAPHIC (GEOG)

[*CIRRIS*, 1986]
[*Knecht*, 1972]

ORIGIN: For rectangular system: center of Earth. For spherical system: no origin (near center of Earth).

ORIENTATION:

X-axis: Points to Greenwich meridian and is in the equatorial plane.

Y-axis: Completes the right-handed orthogonal set, $\overline{Y} = \overline{Z} \times \overline{X}$ and is in the equatorial plane.

Z-axis: Coincident with the Earth's rotational axis and is positive towards north.

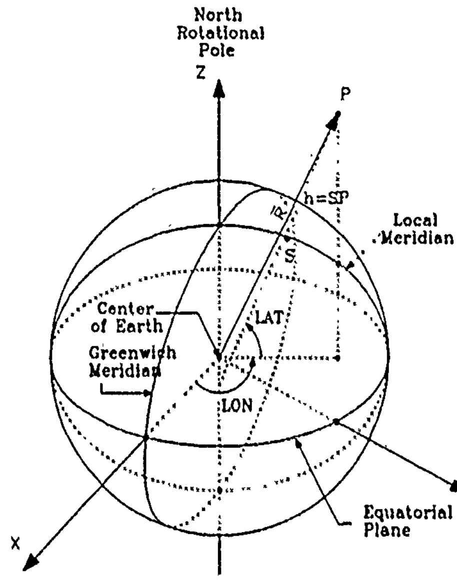

Figure 3-2. GEOD or GEOG System

LAT: Angle measured in the plane of the local meridian from the equatorial plane to the local (ellipsoidal) vertical and is positive towards north.

LON: Angle measured in the equatorial plane from the Greenwich meridian to the local meridian and is positive towards east.

h: Distance from the point of interest P to the reference Fischer ellipsoidal and is measured along the local (ellipsoidal) vertical. It is positive for points outside of ellipsoid. The vector associated with the distance is not parallel to the radial vector $\overline{R}$ from the center of the Earth to the point of interest P.

R: Magnitude of the radius vector $\overline{R}$.

CHARACTERISTICS: Earth-fixed and rotating.

APPLICATIONS: Convenient system for relating the coordinate system of a rotating Earth to inertial and quasi-inertial systems. Used for defining the positions of ground observatories and transmitting and receiving stations.

3.1.3 TOPOCENTRIC EQUATOR (TE) [*Riordan*, 1966]

ORIGIN: A point on the Earth's surface.

ORIENTATION:

A plane parallel to the celestial equator that contains the origin defines the equator reference plane. (E (east), N (north) and R (radial) is also used as an equivalent topocentric system)

X-axis: Lies along the line of intersection of a reference vertical plane, containing either the vernal equinox of epoch, the observer's meridian or the Greenwich meridian, with the equatorial reference plane.

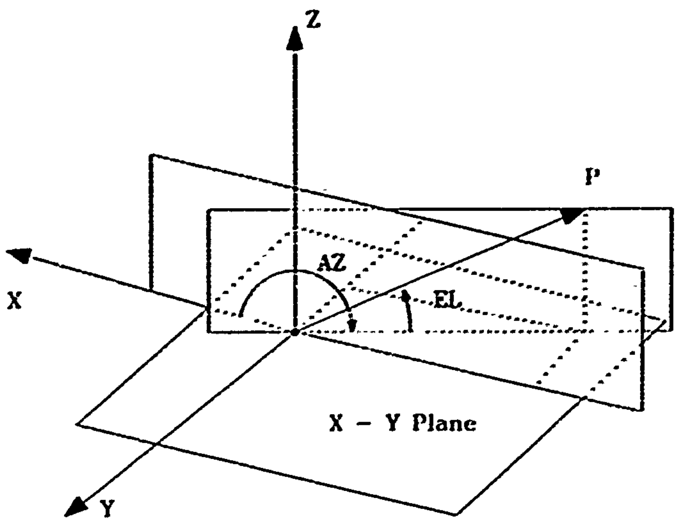

Figure 3-3. TE System

Y-axis: Completes a right-handed orthogonal set, $(\overline{Y} = \overline{Z} \times \overline{X})$.

Z-axis: Perpendicular to the equator reference plane and is positive upward.

X - Y plane: Equatorial reference plane.

EL: Angle of a point of interest P above the equator reference plane.

AZ: Angle measured along the horizon clockwise from a reference vertical plane (normal to the equatorial reference plane and containing either the vernal equinox of epoch, the observer's meridian or the Greenwich meridian) to the vertical plane (normal to the equatorial reference plane) through the point of interest P and the origin.

R: Magnitude of the radius vector $\overline{R}$.

CHARACTERISTICS: Earth-fixed and rotating.

APPLICATIONS: Used for making local observations.

3.1.4 TOPOCENTRIC HORIZON (TH) [*Riordan*, 1966]

ORIGIN: A point on the Earth's surface.

ORIENTATION:
Horizon defines a reference plane that contains the origin.
(E (east), N (north) and R (radial) is also used as an equivalent topocentric system)

X-axis: Lies along the line of intersection of a reference vertical plane (usually containing local north) with the horizon reference plane.

Y-axis: Completes a right-handed orthogonal set, $(\bar{Y} = \bar{Z} \times \bar{X})$.

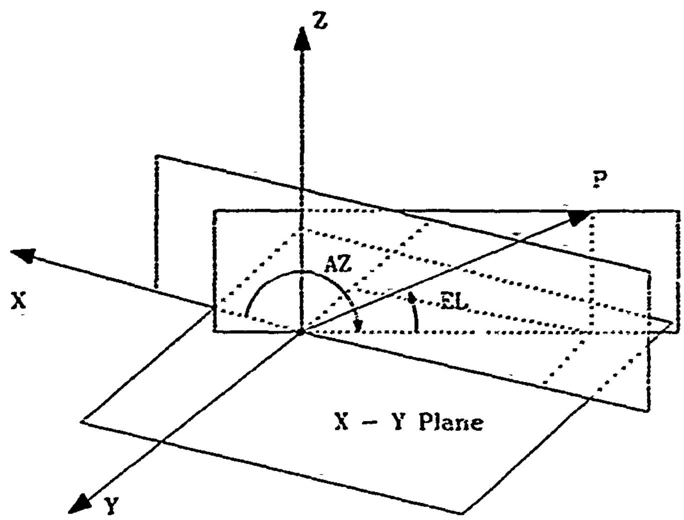

Figure 3-4. TH System

Z-axis: Perpendicular to the horizon reference plane and is positive upward.

X - Y plane: Horizon plane.

EL: Angle of a point of interest P above the horizon reference plane.

AZ: Angle measured along the horizon clockwise from reference vertical plane (normal to the reference plane, usually containing local north) to the vertical plane (normal to the reference plane) through the point of interest P and the origin.

R: Magnitude of the radius vector $\bar{R}$.

CHARACTERISTICS: Earth-fixed.

APPLICATIONS: Experimental measurements are frequently made in this local coordinate system.

3.2 INERTIAL AND QUASI-INERTIAL

3.2.1 EARTH CENTERED INERTIAL (ECI) GEOCENTRIC CELESTIAL INERTIAL (GCI) GEOCENTRIC EQUATORIAL INERTIAL (GEI)

[*CRRES*, 1986]

[*Russell*, 1971]

ORIGIN: Center of Earth.

ORIENTATION:

X-axis: Points in the direction of the first point of Aries (vernal equinox). Axis is in the equatorial and ecliptic planes.

Y-axis: Completes a right-handed orthogonal set, $\overline{Y} = \overline{Z} \times \overline{X}$ and lies in the equatorial plane.

Z-axis: Coincident with the Earth's rotational axis and is positive towards north.

DEC: Angle between the radius vector $\overline{R}$ and the equatorial plane and is positive towards north.

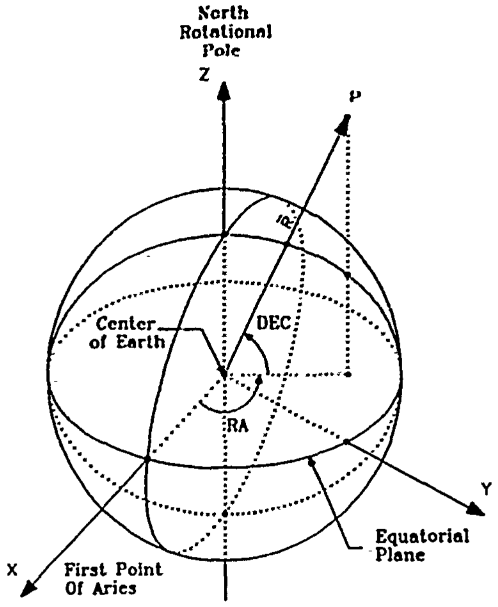

Figure 3-5. ECI, GCI and GEI Systems

RA: Angle between the projection of the radius vector $\overline{R}$ onto the equatorial plane and the vernal equinox. It is positive towards east.

R: Magnitude of the radius vector $\overline{R}$.

3.2.1.1 Reference Epochs for ECI System

The specified epoch establishes the direction for the vernal equinox.

ARIES - TRUE - OF - DATE (ATD) [*CIRRIS*, 1986]
ARIES - MEAN - OF - 1950 (M50)
ARIES - MEAN - OF - 2000 (FK5/J2000)

X-axis: (ATD) - Points towards the first point of Aries for true-of-date.

(M50) - Points towards the mean first point of Aries (vernal equinox) for Epoch 1950.

(FK5/J2000) - Points towards the mean first point of Aries (vernal equinox) for Epoch 2000.

Y-axis: Completes a right-handed orthogonal set, $\overline{Y} = \overline{Z} \times \overline{X}$.

Z-axis: (ATD) - Coincident with rotational axis (true-of-date pole) and is positive towards north.

(M50) - Coincident with mean rotational axis for Epoch 1950 and is positive towards north.

(FK5/J2000) - Coincident with mean rotational axis for Epoch 2000 and is positive towards north.

EP: Equatorial plane of the specified epoch.

CHARACTERISTICS: (ATD) - Quasi-inertial, right-handed, Cartesian system.

(M50, FK5/J2000) - Inertial, right-handed, Cartesian systems.

APPLICATIONS: Used in astronomy, satellite orbit calculations, and to analyze space vehicle dynamics.

3.2.2 Orbital Elements for:

ARIES-TRUE-OF-DATE (ATD) [*CIRRIS*, 1986]
ARIES-MEAN-OF-1950 (M50)
ARIES-MEAN-OF-2000 (FK5/J2000)

ORIGIN: Center of Earth.

ORIENTATION:

The reference for computing osculating orbital elements is the Aries-true-of-date, the 1950, or the 2000 coordinate systems.

a: semi-major axis of the orbit.

e: eccentricity of the orbit.

INC: angle between Earth's rotational axis and the orbital angular momentum vector.

RA: angle measured eastward from the vernal equinox in the equatorial plane to that intersection with the orbit plane where the vehicle passes from south to north. In the case where inclination equals zero, the ascending node is defined to be the X-axis of the reference system.

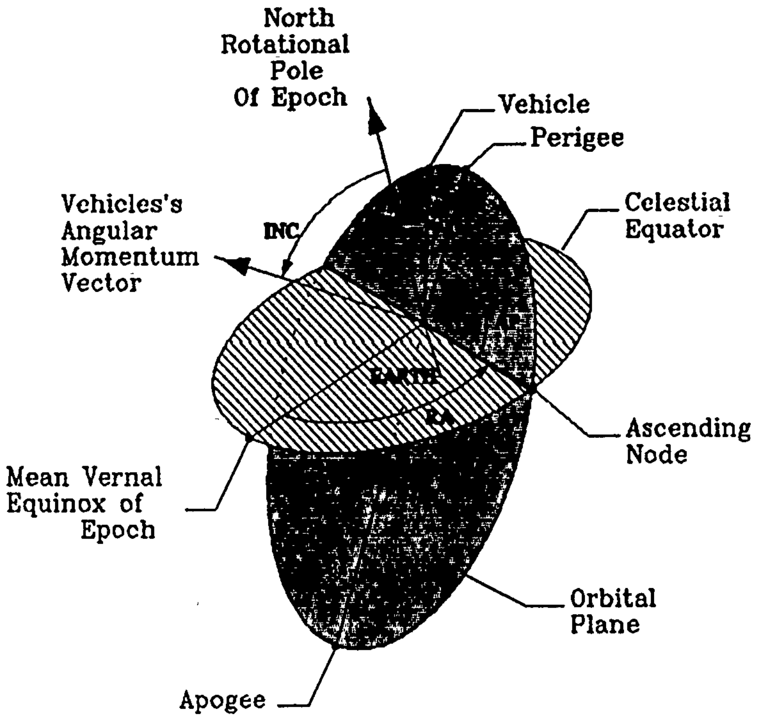

Figure 3-6. ATD, M50, and FK5/J2000 Orbital Elements

AP: angle measured in the orbit plane between the ascending node and perigee, positive in the direction of travel in the orbit. In the case where eccentricity equals zero, perigee is defined to be at the ascending node.

TA: geocentric angular displacement of the vehicle measured from perigee in the orbit plane and positive in the direction of travel in the orbit. Mean Anomaly is usually provided instead.

CHARACTERISTICS: (ATD): Quasi-Inertial.

(M50, FK5/J2000): Inertial.

APPLICATIONS: Commonly used in satellite orbit calculations.

3.2.3 Polar Velocity Coordinates for:
ARIES-TRUE-OF-DATE (ATD)
ARIES-MEAN-OF-1950 (M50)
ARIES-MEAN-OF-2000 (FK5/J2000)
GREENWICH-TRUE-OF-DATE, GEOGRAPHIC (GEOG)

[*CIRRIS*, 1986]

ORIGIN: Point of interest, P(x, y, z).

ORIENTATION:

U, E and N denote up, east and north directions, then:

ψ, azimuth: angle from north to the projection of $\overline{V}$ on the reference plane, positive towards east.

γ, flight-path angle: angle between the reference plane and the velocity vector $\overline{V}$ (positive towards U).

V: magnitude of velocity vector $\overline{V}$.

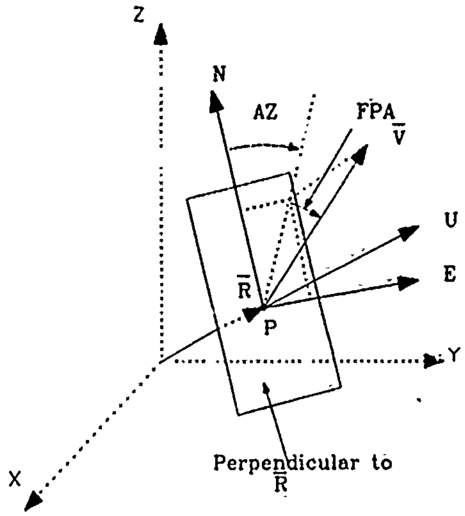

Figure 3-7. Polar Velocity Coordinates for ATD, M50, FK5/J2000 and GEOG

CHARACTERISTICS:

(ATD): Velocity system associated with Aries-true-of-date system, quasi-inertial.

(M50, FK5/J2000): Velocity system associated with Aries-mean-of-1950 (2000) system, inertial.

(GEOG): Velocity system associated with Greenwich-true-of-date (Geographic) system, quasi-inertial.

APPLICATIONS:

(ATD, M50 and FK5/J2000): Used in satellite velocity calculations.

(GEOG): Used in satellite orbit and velocity related calculations.

3.2.4 ATS-1 (ATS) [*Russell*, 1971]

ORIGIN: Center of the Earth.

ORIENTATION:

X-axis: Points to local noon or sub-solar point meridian, and lies in the equatorial plane.

Y-axis: Completes a right-handed orthogonal set, $\overline{Y} = \overline{Z} \times \overline{X}$. Perpendicular to the Earth-Sun line towards dusk and is in the equatorial plane.

Z-axis: Coincident with the Earth's rotational axis and is positive towards north.

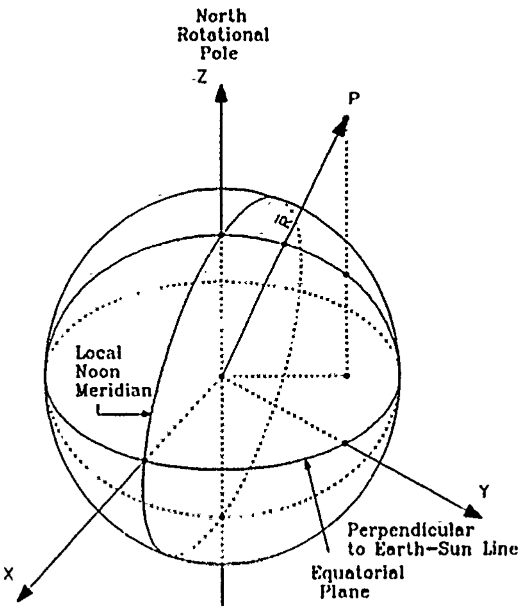

Figure 3-8. ATS System

CHARACTERISTICS:

Geocentric.

APPLICATIONS: Used extensively in the analysis of magnetometer data from the Applications Technology Satellite-1 (ATS-1) satellite.

3.2.5 ECLIPTIC (ECL) SOLAR ECLIPTIC (SE) GEOCENTRIC SOLAR ECLIPTIC (GSE)

[*Riordan*, 1966]
[*Knecht*, 1972]
[*Russell*, 1971]

ORIGIN: Center of Earth.

ORIENTATION:

X-axis: Points to Sun and is in the ecliptic plane.

Y-axis: Completes a right-handed orthogonal set, $\overline{Y} = \overline{Z} \times \overline{X}$.

Z-axis: Perpendicular to the ecliptic plane (parallel to the ecliptic pole) and is positive towards north.

LAT: Angle measured at the center of the Earth between the radius vector $\overline{R}$ and the ecliptic plane.

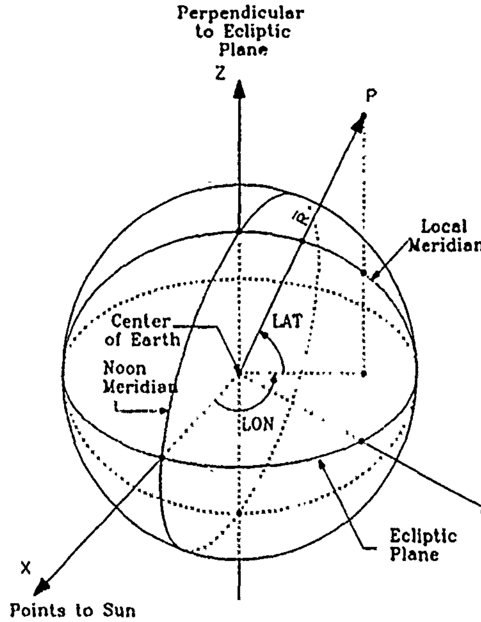

Figure 3-9. ECL, SE and GSE Systems

LON: Angle measured eastward along the intersection of the ecliptic plane with the Earth's surface from the noon meridian to the local meridian containing the radius vector $\overline{R}$.

R: Magnitude of the radius vector $\overline{R}$.

CHARACTERISTICS: (ECL and SE): Geocentric and quasi-inertial. Relative to an inertial system, ECL and SE systems have a yearly rotation.

(GSE): Geocentric and quasi-inertial.

APPLICATIONS: Used to display satellite trajectories, interplanetary magnetic field observations and solar wind velocity data.

3.2.6 GEOCENTRIC SOLAR EQUATORIAL (GSEQ) [*Russell*, 1971]

ORIGIN: Center of Earth.

ORIENTATION:

X-axis: Points toward the Sun and is in the ecliptic plane.

Y-axis: Completes a right-handed orthogonal set, $\overline{Y} = \overline{Z} \times \overline{X}$, and is parallel to the solar equatorial plane and points towards dusk.

Z-axis: In plane containing X-axis and Sun's rotational axis, and is positive towards north.

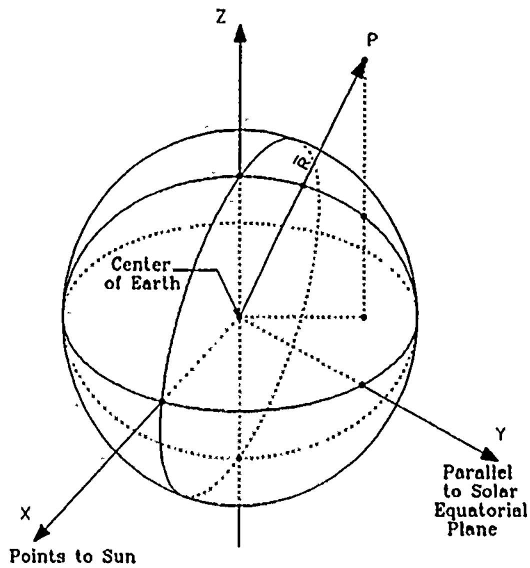

Figure 3-10. GSEQ System

CHARACTERISTICS: Geocentric, quasi-inertial, right-handed, Cartesian system.

APPLICATIONS: Used to display interplanetary magnetic field

3.2.7 PQW [*Escobal*, 1965]

ORIGIN: Center of the Earth.

ORIENTATION:

Defined by orbit of any particular satellite, quasi-inertial.

X-axis (also called P):
Points toward perigee and is in the plane of the orbit.

Y-axis (also called Q):
Completes right-handed orthogonal set $\overline{Y} = \overline{Z} \times \overline{X}$, and lies in the orbital plane. Advanced 90° from P in the direction of motion of the satellite.

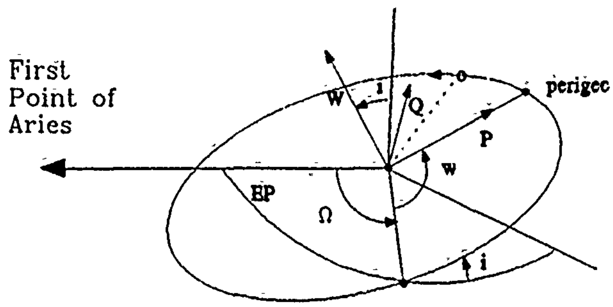

Figure 3-11. PQW System

Z-axis (also called W):
The positive orbital plane normal; at the inclination angle i relative to Earth's north rotational axis.

EP: Equatorial plane of ECI system.

CHARACTERISTICS: Geocentric, quasi-inertial, right-handed, Cartesian system. Gradually reorients relative to ECI system as right ascension of ascending node **Ω**, argument of perigee w, and inclination i progress due to terrestrial and luni-solar perturbations. The classical two-dimensional Keplerian orbital ellipse is defined in this system.

APPLICATIONS: Valuable intermediate coordinate system for many orbit-related calculations, such as eclipsing and station viewing.

3.3 MAGNETIC

3.3.1 CENTERED DIPOLE (D)

[*Egeland*, 1973]

ORIGIN: Center of Earth.

ORIENTATION:

X-axis: Points in the direction of the geographic meridian containing the north magnetic dipole pole (or equivalently, the south geographic pole) and is perpendicular to the magnetic dipole axis.

Y-axis: Completes the right-handed orthogonal set, $\overline{Y} = \overline{Z} \times \overline{X}$.

Z-axis: Coincident with the magnetic dipole (geomagnetic) axis and is positive towards north. For Epoch 1990, the north magnetic dipole pole has geographic coordinates of 79.19° north, 289.02° east (70.98° west) and the south magnetic dipole pole has geographic coordinates of 79.19° south and 109.02° east.

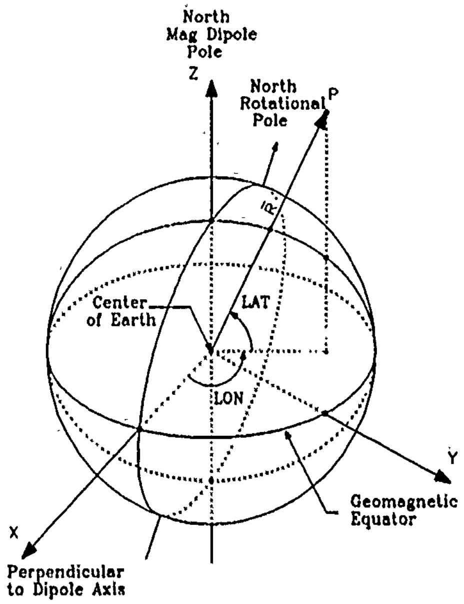

Figure 3-12. DIPOLE (D) System

LAT: Geocentric angle measured from the geomagnetic equator to the radius vector $\overline{R}$ along the geomagnetic meridian containing $\overline{R}$ (positive towards north).

LON: Geocentric angle measured from the geographic meridian containing the north magnetic dipole (or south geographic) pole to the projection of the radius vector $\overline{R}$ onto the geomagnetic equator and is positive towards east.

R: Magnitude of the radius vector $\overline{R}$.

CHARACTERISTICS: Geocentric, Earth-fixed and rotating.

APPLICATIONS: Used to study the geomagnetic field in close proximity to Earth and for simple analytical Earth-centered magnetic field model.

3.3.2 MAGNETIC (MAG) ECCENTRIC DIPOLE alternatively, DIP POLE

[*Chapman*, 1940]

ORIGIN:

Near the center of the Earth.

ORIENTATION:

X-axis: Perpendicular to the magnetic axis and parallel to the Centered Dipole X-axis. See previous page.

Y-axis: Completes the orthogonal set, $\overline{Y} = \overline{Z} \times \overline{X}$.

Z-axis: For Eccentric Dipole, parallel to the Centered Dipole Z-axis. See previous page.
For Dip Pole, parallel to the magnetic axis (magnetic dip poles) and is positive towards north. For Epoch 1990, north magnetic dip pole has geographic coordinates of 77.8° north and 103.7° west and south magnetic dipole has the geographic coordinates of 64.9° south and 139.4° east.

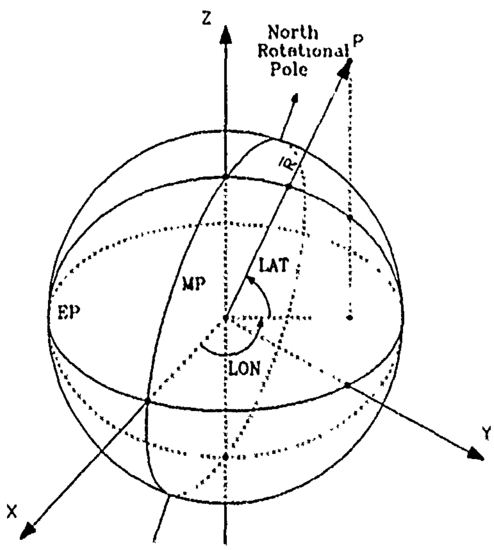

Figure 3-13. MAG System

MP: Meridional plane defined by the magnetic axis and the south geographic pole.

EP: Magnetic equatorial plane.

LAT: Geocentric angle measured from the projection of the radius vector $\overline{R}$ onto the magnetic equatorial plane to the radius vector $\overline{R}$ (positive northward of magnetic equatorial plane).

LON: Geocentric angle measured along the magnetic equator from the meridian that is defined by the magnetic axis and the south geographic pole to the local magnetic meridian that contains the radius vector $\overline{R}$.

R: Magnitude of the radius vector $\overline{R}$.

CHARACTERISTICS: Offset from geocentric due to the spherical asymmetry of the geomagnetic field. Various definitions possible.

APPLICATIONS: Used often for determining the position of magnetic observations and in navigation. Also used in magnetic field line tracing.

3.3.3 DIPOLE MERIDIAN (DM) [*Russell*, 1971]

ORIGIN: Center of Earth.

ORIENTATION:

X-axis: Perpendicular to the geomagnetic (magnetic) dipole axis and lies in the geomagnetic equatorial plane. Completes the right-handed orthogonal set,

Y-axis: Perpendicular to a radius vector to the point of interest and lies in the geomagnetic equatorial plane and is positive towards east.

Z-axis: Coincident with the geomagnetic (magnetic) dipole axis and is positive towards north. For Epoch 1990, the north magnetic dipole pole has geographic coordinates of 79.19° north and 289.02° east (70.98° west), and the south magnetic dipole pole has the geographic coordinates of 79.19° south and 109.02° east.

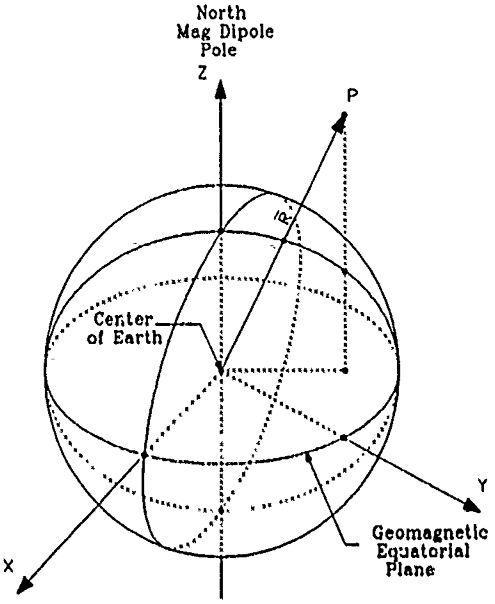

Figure 3-14. DM System

CHARACTERISTICS: Geocentric, Earth-fixed and rotating.

APPLICATIONS: Used to order data controlled by the dipole magnetic field where the solar wind interaction with the magnetosphere is weak. Used to describe distortions of the magnetic field in terms of the two angles, declination and inclination.

3.3.4 GEOCENTRIC SOLAR MAGNETOSPHERIC (GSM) SOLAR MAGNETOSPHERIC (SMC)

[*CRRES*, 1986; *Russell*, 1971] [*Bass*, 1977; *Knecht*, 1972]

ORIGIN: Centered, or optionally, Eccentric Dipole.

ORIENTATION:

X-axis: Points to the Sun.

Y-axis: Perpendicular to the geomagnetic (magnetic dipole) axis and is positive towards dusk. Axis lies in the geomagnetic equatorial plane and completes an orthogonal set, $\overline{Y} = \overline{Z} \times \overline{X}$.

Z-axis: Perpendicular to the X-axis and in the plane containing the X-axis and the magnetic dipole (geomagnetic) axis. It is positive towards north.

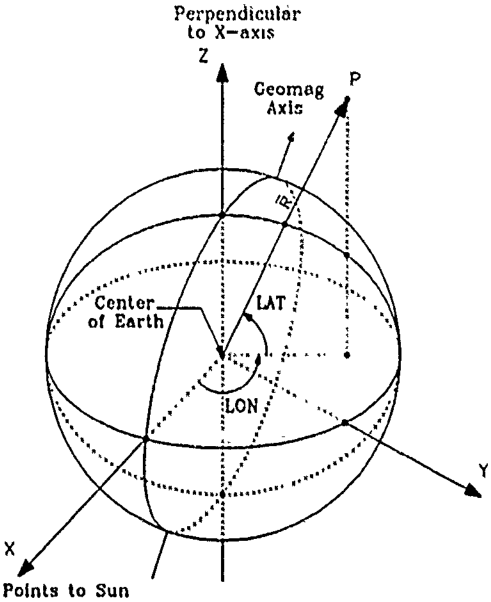

Figure 3-15. GSM and SMC Systems

LAT: Angle measured from the projection of the radius vector $\overline{R}$ onto the X - Y plane to the radius vector $\overline{R}$.

LON: Angle measured in the X - Y plane from the X-axis to the projection of $\overline{R}$ onto the X - Y plane and is positive towards east.

R: Magnitude of radius vector $\overline{R}$.

CHARACTERISTICS: Geocentric. System rocks about the solar direction on yearly (23.4° ± 11.2°) and 24 hour (± 11.2°) cycles.

APPLICATIONS (GSM): Useful for displaying magnetopause and shock boundary positions, magnetosheath and magnetotail magnetic fields and magnetosheath solar wind velocities because the orientation of the magnetic dipole axis alters the otherwise cylindrical symmetry of the solar wind flow. Useful in models of magnetopause currents. Used to examine the statistical relationships of geomagnetic activity to inter-planetary magnetic field (IMF) parameters.

(SMC): Useful in areas of magnetospheric physics such as the interaction of the solar wind with the Earth's magnetic field.

3.3.5 SOLAR MAGNETIC (SM) SOLAR GEOMAGNETIC (SG)

[*Russell*, 1971]

ORIGIN: Centered, or optionally, Eccentric Dipole.

ORIENTATION:

X-axis: Perpendicular to the Z-axis and is in the plane containing the Z-axis and the Sun (positive towards the Sun). Axis does not necessarily point at the Sun but does lie in the geomagnetic equatorial plane.

Y-axis: Perpendicular to the Earth-Sun line (positive towards dusk) and lies in the geomagnetic equatorial plane. Completes orthogonal set, $\overline{Y} = \overline{Z} \times \overline{X}$.

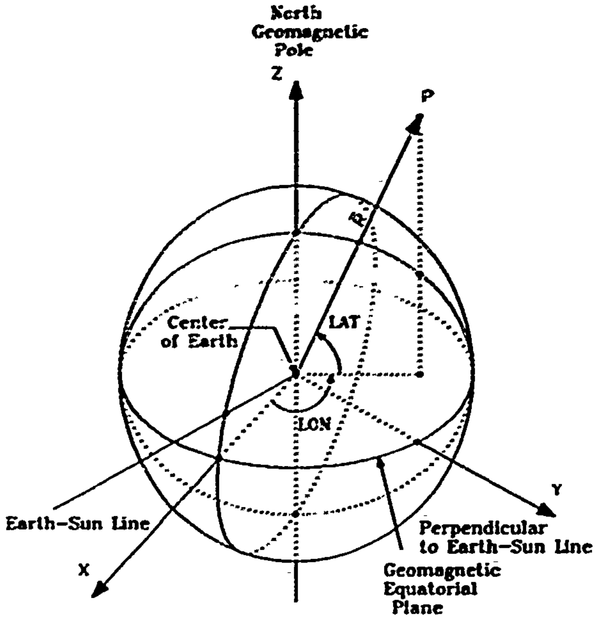

Figure 3-16. SM and SG Systems

Z-axis: Coincident with the geomagnetic (magnetic dipole axis) and is positive towards north.

LAT: Geocentric angle measured from the projection of the radius vector $\overline{R}$ onto the geomagnetic equatorial plane to the radius vector $\overline{R}$ and is positive towards north.

LON: Geocentric angle measured in the geomagnetic equatorial plane from the geomagnetic meridian containing the Earth-Sun line to the geomagnetic meridian containing the radius vector $\overline{R}$ and is positive towards east.

R: Magnitude of the radius vector $\overline{R}$.

CHARACTERISTICS: Geocentric. System rotates with both a yearly and daily period with respect to inertial coordinates.

APPLICATIONS: Useful for ordering data controlled more strongly by the Earth's dipole field than by the solar wind. It has been used for magnetopause cross-sections and magnetospheric magnetic fields.

3.3.6 SOLAR WIND MAGNETOSPHERIC (SWM)

ORIGIN: Center of the Earth.

ORIENTATION:

X-axis: Points to the solar wind. The axis is positive in the direction opposite to the solar wind.

Y-axis: Perpendicular to the geomagnetic axis (magnetic dipole) and is positive towards dusk. The axis lies in the geomagnetic equatorial plane and completes an orthogonal set, $\overline{Y} = \overline{Z} \times \overline{X}$.

Z-axis: Perpendicular to the X-axis and is in the plane containing the X-axis and the geomagnetic axis (magnetic dipole) and is positive towards north.

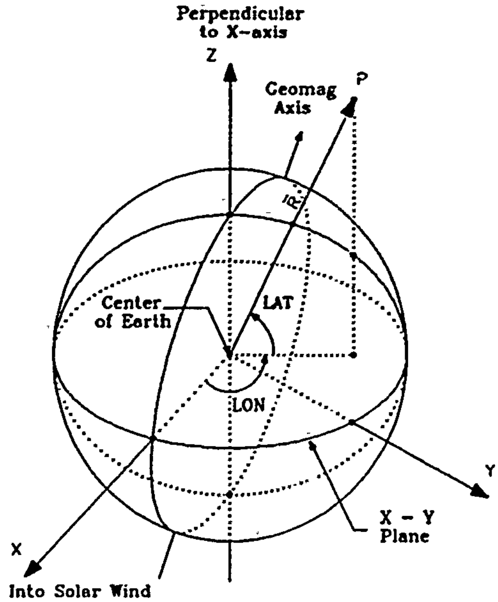

Figure 3-17. SWM System

LAT: Geocentric angle from the projection of the radius vector $\overline{R}$ onto the X - Y plane to the radius vector $\overline{R}$ and is positive towards north.

LON: Geocentric angle measured in the X - Y plane from the plane containing the X and Z axes to the plane containing the Z-axis and the radius vector $\overline{R}$.

R: Magnitude of the radius vector $\overline{R}$.

CHARACTERISTICS: Geocentric, aberrated.

APPLICATIONS: Useful in studying the effects of the solar wind on the magnetosphere.

3.3.7 SOLAR WIND (SW)

ORIGIN: Center of the Earth.

ORIENTATION:

X-axis: Points to the solar wind. The axis lies in the ecliptic plane and is positive in the direction opposite to the solar wind.

Y-axis: Completes a right-handed orthogonal set, $\overline{Y} = \overline{Z} \times \overline{X}$ and the axis lies in the ecliptic plane.

Z-axis: Perpendicular to the ecliptic plane and is positive towards north.

LAT: Geocentric angle measured from the projection of the radius vector $\overline{R}$ onto the ecliptic plane to the radius vector $\overline{R}$.

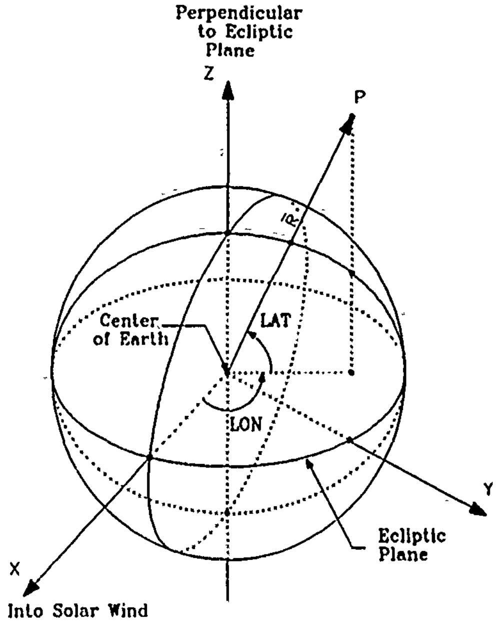

Figure 3-18. SW System

LON: Geocentric angle measured from the plane containing the X and Z axes to the plane containing the Z-axis and the radius vector $\overline{R}$.

R: Magnitude of the radius vector $\overline{R}$.

CHARACTERISTICS: Geocentric, aberrated.

APPLICATIONS: Useful in analyzing the impact of the solar wind on hemispheric events.

3.3.8 MAGNETIC FIELD (MFD) [*McInerney*, 1973]

ORIGIN: Point of interest.

ORIENTATION:

X-axis: Component of the magnetic field in the geographic north direction.

Y-axis: Component of the magnetic field in the geographic east direction.

Z-axis: Vertical component of the magnetic field and is positive downwards.

INC: (or DIP): Angle measured from the horizontal component, $\overline{H_z}$, of the magnetic field to the total magnetic field vector $\overline{B}$ and is positive downwards.

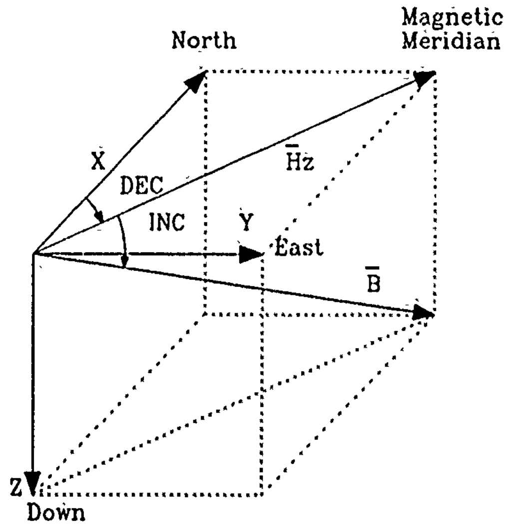

Figure 3-19. MFD System

DEC: Angle between the north component of the magnetic field (X-axis) and the horizontal component ($\overline{H_z} = \overline{X} + \overline{Y}$ components) of the magnetic field. It is positive towards east.

B: Magnitude of the magnetic field vector $\overline{B}$.

CHARACTERISTICS: Fixed at the point of interest.

APPLICATIONS: Used in analysis of the components of the geomagnetic field.

3.3.9 VDH

ORIGIN: Point of interest.

ORIENTATION:

X-axis (also called V):
Completes an orthogonal set, $\overline{X} = \overline{Y} \times \overline{Z}$, $(\overline{V} = \overline{D} \times \overline{H})$.

Y-axis (also called D):
Perpendicular to the radius vector $\overline{R}$ and the Z-axis and points towards east.

Z-axis (also called H):
Parallel to the magnetic dipole axis and is positive towards the north.

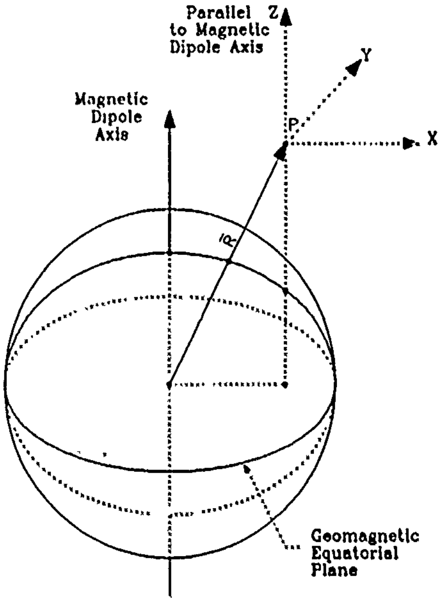

Figure 3-20. VDH System

CHARACTERISTICS:

Right-handed, Cartesian system.

APPLICATIONS: Used in analysis of geomagnetic field data. Convenient to use because magnetic field lines lie mostly within the X - Z plane which allows the magnitude of the magnetic field component along the Y-axis to approach zero.

3.4 OTHER COORDINATE SYSTEMS

3.4.1 BODY AXIS (BA) LOOK ANGLE (LA)

[*CIRRIS*, 1986]

ORIGIN: Center of gravity (mass).

ORIENTATION:

X-axis: Parallel to the Orbiter structural body axis (positive towards the nose).

Y-axis: Completes the right-handed orthogonal set, $\overline{Y} = \overline{Z} \times \overline{X}$.

Z-axis: Parallel to the Orbiter plane of symmetry and is perpendicular to the X-axis (positive down with respect to the Orbiter fuselage).

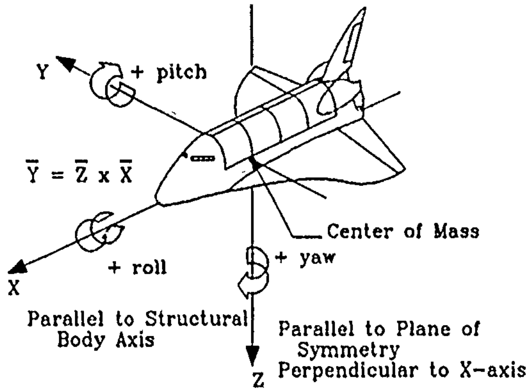

Figure 3-21. BA and LA Systems

CHARACTERISTICS:

Rotating, right-handed, Cartesian system.

The Euler sequence that is commonly associated with this system is a yaw, pitch, roll sequence where ψ = yaw, θ = pitch and ϕ = roll or bank. This attitude sequence is yaw, pitch and roll around the Z, Y and X axes, respectively.

Look angles define a vector direction relative to the body axes of the vehicle. Since only direction is involved, the origin is translatable to any point on the body.

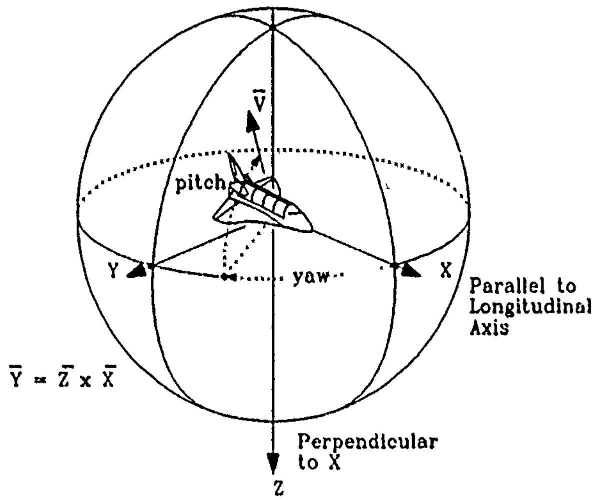

Figure 3-22. LA System

$\overline{V}$ is a vector line of sight.

YAW: measured from the X-axis to the projection of $\overline{V}$ into the X - Y plane and is a right-handed rotation about the Z-axis from 0° to 360°.

PITCH: the angle between $\overline{V}$ and its projection into the X - Y plane and is formed by a right-handed rotation about the previously rotated Y-axis. It is positive towards the -Z-axis.

A unit vector along the X-axis will be made to coincide with $\overline{V}$ by rotating the vector through the yaw angle about the Z-axis, then through the pitch angle about the Y-axis, in that order. Y' is Y location resulting from the first location. This coordinate system can be used as an instrument pointing coordinate system when the origin is translated to the instrument center location. A third rotation, analogous to body roll, is used to establish the instrument index referenced to the body axis coordinate system.

CHARACTERISTICS: Rotating, right-handed, Cartesian system.

APPLICATIONS: Commonly used to orient Orbiter into desired positions. Direction of celestial objects, ground stations, other orbiting vehicles, etc. from the Orbiter can be reported in this system. It may be used as a basis for defining Orbiter-antenna radiation-distribution patterns.

3.4.2 LOCAL VERTICAL LOCAL HORIZONTAL (LVLH) LOCAL ORBITAL (LO)

[*CIRRIS*, 1986]

ORIGIN: Vehicle center of mass.

ORIENTATION:

The X-axis and Z-axis form the instantaneous orbit plane at the time of interest.

X-axis: It lies in the vertical orbital plane, perpendicular to the Z-axis and is positive in the direction of the vehicle motion.

Y-axis: Perpendicular to the orbital plane and completes a right-handed orthogonal set, $\overline{Y} = \overline{Z} \times \overline{X}$.

Z-axis: It lies along the geocentric radius vector to the vehicle and is positive towards the center of the Earth.

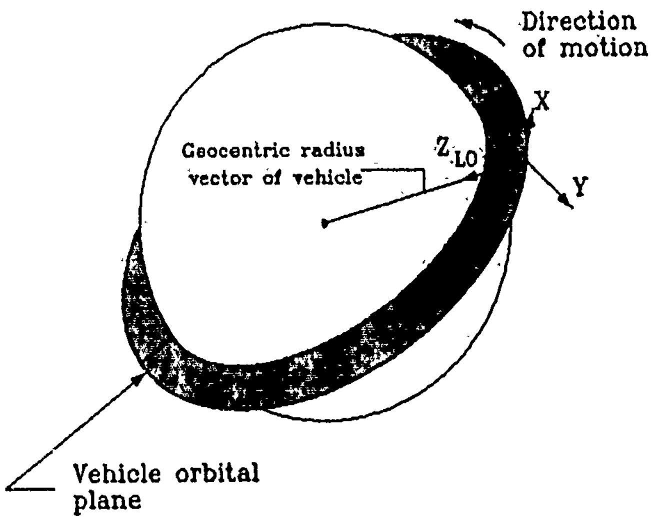

Figure 3-23. LVLH, LO Systems

CHARACTERISTICS

(BOTH): Quasi-inertial, right-handed, Cartesian system.

APPLICATIONS (BOTH): Useful in converting orbital orientation to fixed-Earth orientation.

3.4.3 UVW (U) [*CIRRIS*, 1986]

ORIGIN: Point of interest.

ORIENTATION:

The X and Y axes form the instantaneous orbit plane at epoch.

X-axis (also called U): It lies along the geocentric radius vector to the vehicle and is positive radially outward.

Y-axis (also called V): Completes an orthogonal set $\overline{Y} = \overline{Z} \times \overline{X}$ (*also* $\overline{V} = \overline{W} \times \overline{U}$). Thus, $\overline{V}$ is generally in the direction of, but not coincident with, the velocity vector.

Z-axis (also called W): It lies along the instantaneous orbital angular momentum vector at epoch and is positive in the direction of the angular momentum vector.

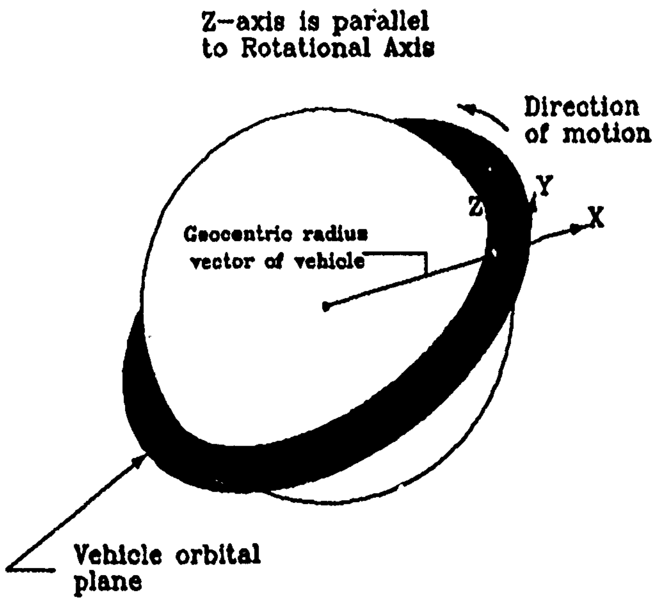

Figure 3-24. UVW (U) System

CHARACTERISTICS:

Quasi-inertial, right-handed, Cartesian system. This system is quasi-inertial in the sense that it is treated as an inertial coordinate system but it is redefined at each point of interest.

APPLICATIONS: Useful in converting orbital orientation to fixed-Earth orientation.

3.4.4 SENSOR (S) [*Bass*, 1977]

ORIGIN: Predefined position in the sensor.

ORIENTATION:

X and Y axes: Are in predefined sensor plane and form a right-handed orthogonal system with the Z-axis, which is usually the line of sight.

$\overline{V}$: Vehicle velocity vector.

ALPHA, α: Angle measured from the X-axis to the projection of the velocity vector $\overline{V}$ onto the X - Y plane.

BETA, β: Angle measured from the Z-axis to the velocity vector $\overline{V}$.

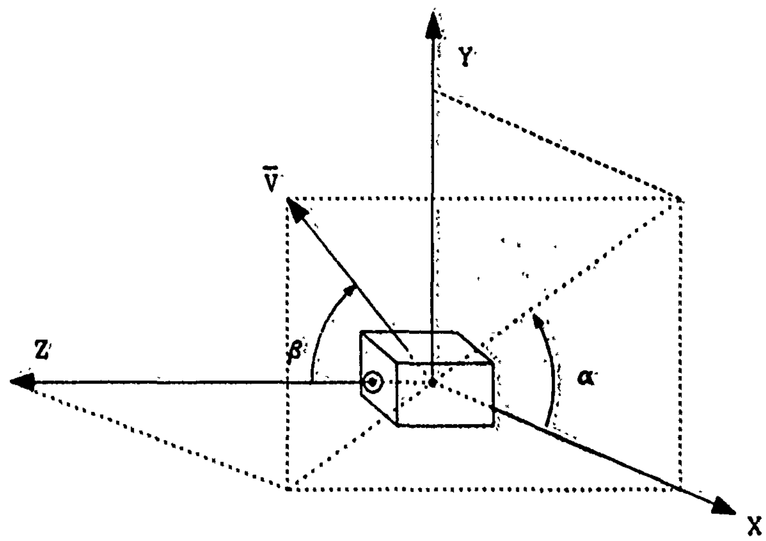

Figure 3-25. SENSOR (S) System

CHARACTERISTICS:

Sensor-fixed.

APPLICATIONS: Used as a foundation for converting measured sensor data to other coordinate systems.

3.4.5 PLASMA (P) [*Egeland*, 1973]

ORIGIN: Point of interest.

ORIENTATION:

X-axis: Perpendicular to the Z-axis, lies in a plane defined by the Z-axis and the propagation vector $\overline{W}$.

Y-axis: Completes a right-handed orthogonal set, $\overline{Y} = \overline{Z} \times \overline{X}$.

Z-axis: Parallel to the magnetic field and is positive in the same sense as the magnetic field.

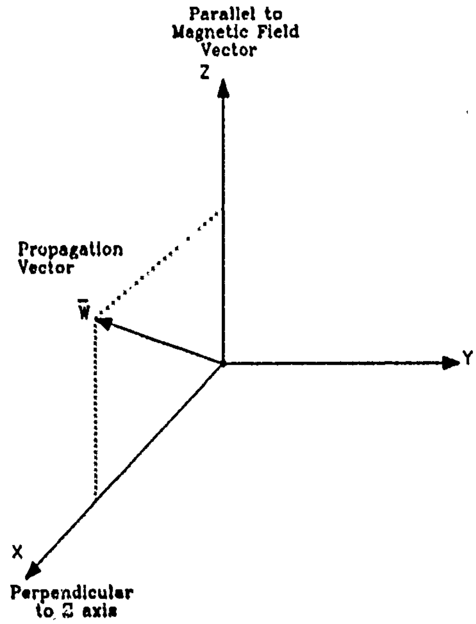

Figure 3-26. PLASMA (P) System

CHARACTERISTICS:

Right-handed, Cartesian system.

APPLICATIONS: Used to study wave propagation in plasmas.

3.4.6 WAVE (W)

[*Egeland*, 1973]

ORIGIN: Point of interest.

ORIENTATION:

X-axis: Completes an orthogonal set, $\overline{X} = \overline{Y} \times \overline{Z}$.

Y-axis: Perpendicular to the Z-axis and lies in a plane defined by the Z-axis and the magnetic field vector $\overline{B}$. It is positive in the same sense as $\overline{B}$.

Z-axis: Parallel to the wave propagation vector and is positive in the direction of the wave propagation.

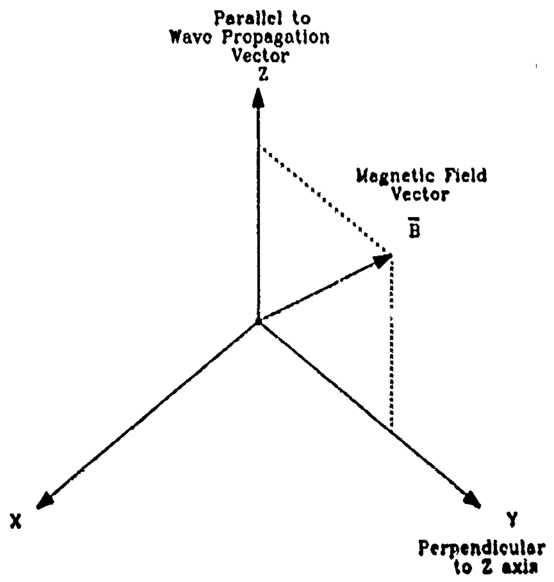

Figure 3-27. WAVE (W) System

CHARACTERISTICS:

Right-handed, Cartesian system.

APPLICATIONS: Used to study wave propagation.

3.5 OTHER MAGNETIC COORDINATE SYSTEMS

3.5.1 GEOMAGNETIC COORDINATES B AND L

ORIGIN: Not applicable.

ORIENTATION: (See Figure 3-28).

B - The strength of the magnetic field at the point of interest.

L - a parameter which characterizes the magnetic field line passing through the point of interest and/or trapped particles mirroring at said point. The e several definitions in use which are equivalent for a dipole field:

L_m or L-shell - the definition given by McIlwain [1961] - the equatorial radius, in Earth radii, of a dipole field line passing through a point with the same longitudinal invariant integral I is the familiar integral appearing in the second adiabatic invariant:

$$I = \int \left[1 - \frac{B(s)}{B_m}\right]^{1/2} ds$$

where the integration is along the field line between the point of interest and its conjugate point; B(s) is the magnetic field strength at a point along the field line, and B_m is the field strength at the point of interest (same as B above). This definition can not be formulated analytically. McIlwain [1961,1966] and Hilton [1971] have given analytic approximations for L_m as a function of I and B. Stern [1968] has given an approximation for L_m for perturbed dipole fields, good to first order in the perturbation, which circumvents the need to compute I. However, this approximation is valid only for fields expressible as the gradient of a potential, i. e., for fields in current-free environments. The perturbation approach could possibly be generalized for application to more arbitrary fields.

L_e - the equatorial radius of the actual field line passing through the point of interest [*Stone*, 1963].

L_0 - the McIlwain L value at the equatorial crossing point of the field line passing through the point of interest [*Stone*, 1963].

L_d - the equatorial radius of the dipole field line which the actual field line through the point of interest approaches in the high latitude limit [*Schulz and Lanzerotti*, 1974].

CHARACTERISTICS: The L_m and B of trapped particle's mirroring points are adiabatically conserved for a stationary magnetic field. Stone [1963] has found relations between the variation of L_m along a field line and the extent of shell-splitting between particles mirroring at different points along the same field line at a particular longitude. The L_m definition fails when internal multipoles are included, since the field does not approach a dipole in the high latitude limit. This definition, when applicable, and the other two definitions, L_e and L_0, are pure properties of a field line which differ only slightly from L_m in the inner radiation belt.

APPLICATIONS: L_m and B are useful for organizing trapped particle data because of the conservation property mentioned above and the close relation of L_m to the field lines along which the particles are napped. Since this property is independent of longitude, particle fluxes in stationary fields can be expressed as functions of these two variables alone.

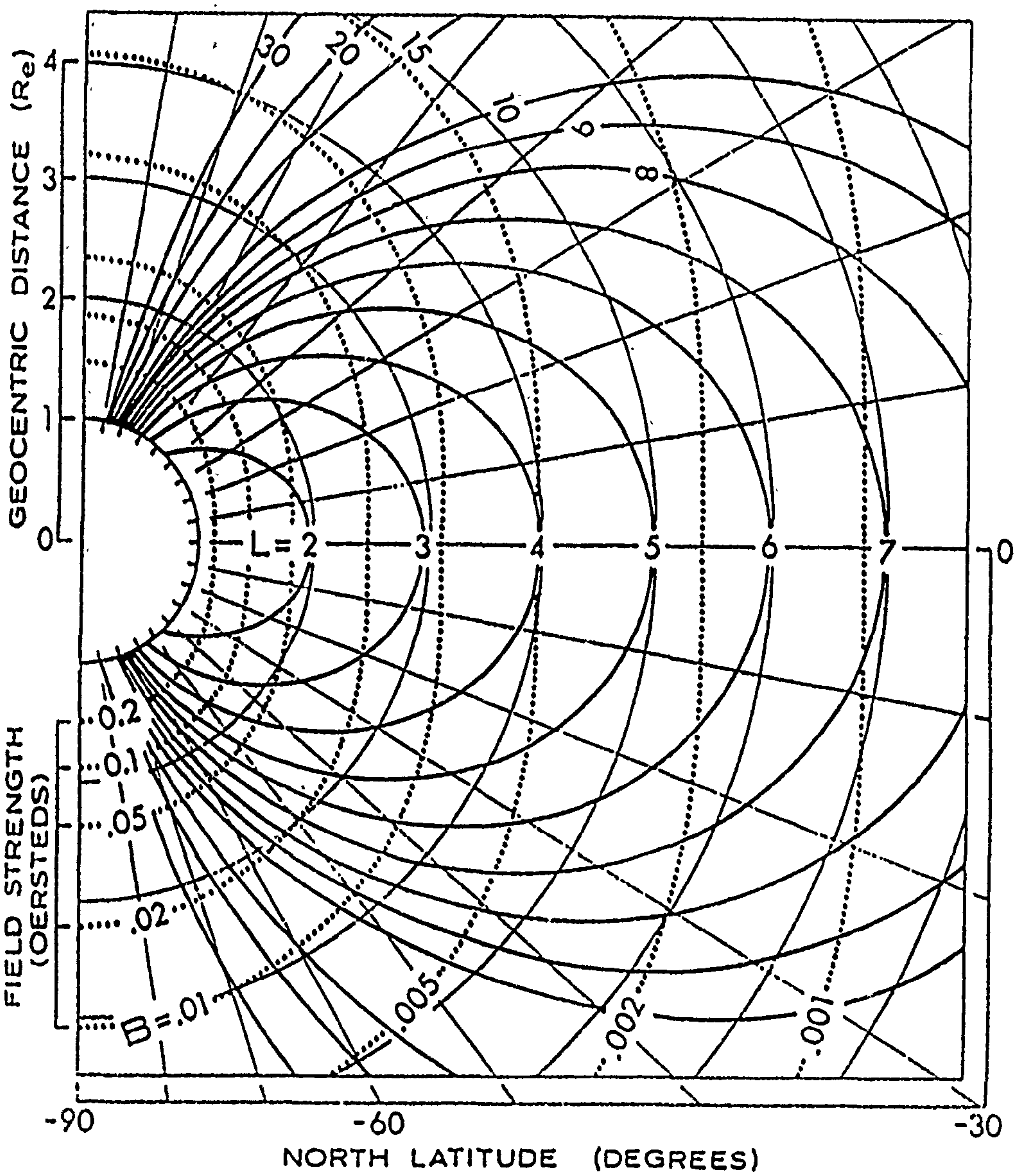

Figure 3-28. B and L System. The curves are the intersection with a magnetic meridian plane of surfaces of constant B and L. The departure of these curves, plotted for a dipole field, from those of the actual field is too small to be apparent in a figure of this scale [*Knecht*, 1972].

3.5.2 CORRECTED GEOMAGNETIC (CGEOM)

ORIGIN: Not applicable.

ORIENTATION:

A different class of geomagnetic coordinate systems and conversions arise when the asymmetrical geomagnetic field of Earth is considered. The Corrected Geomagnetic coordinate system was introduced to improve the accuracy of the Geomagnetic coordinate system. The Geomagnetic system is based only on a simple centered dipole. The Corrected Geomagnetic system uses spherical harmonic analysis to establish a more accurate specification of a geomagnetic field model. Hakura and Gustafsson [1970] have utilized a procedure to trace magnetic field lines to the geomagnetic equator, following which the equivalent geomagnetic dipole latitude and longitude, or Corrected Geomagnetic Coordinates, are obtained in tabular form, suitable for interpolation. This system should be used only for altitudes near Earth since the tables are typically derived from ground-based transformations.

APPLICATION: Used in the analysis of the distortion of the magnetic field in close proximity to the Earth's surface.

3.6 TIME MEASUREMENT SYSTEMS

The meaning of time varies depending upon how this measure is applied. When an epoch value can be identified with the data, the most accepted measure is Universal Time (UT), or Greenwich Mean Time (GMT), which corresponds to the mean hour angle of Greenwich, or 0° longitude, relative to the Sun.

To accurately determine the relative positions of the Sun, moon, planets and Earth rotation, Newton's equations of motion must be applied to model the celestial dynamics, and for this, a uniform measure of time, called Terrestrial Dynamic or Ephemeris Time (TDT or ET) which is the independent argument in those equations, must be used. ET differs slightly from UT, and the adjustment ΔT that must be made to UT to obtain ET is actually fixed only after accurate ephemerides of the celestial bodies are known following observations of their positions. The history and the projections for this additive constant which drifts about one second per year, are published in the Astronautical Almanac [1990].

To obtain a fixed reference on the celestial sphere, one can measure the Earth's rotation from the vernal equinox. This measure is called Sidereal time. It should be noted that the universal day is about 237 seconds longer than the sidereal day.

Another measure of time that is important comes about from the fact that ionospheric processes turn out to be ordered in solar/geomagnetic coordinates called Geomagnetic Local Time (MLT). This value is assumed to be distributed uniformly with geomagnetic longitude, 1 hour per 15°, where 0 hr MLT is defined to be at the antipode of the sub-solar point. Thus, if ML_{sat} is the magnetic longitude of the satellite and ML_{ant} is the magnetic longitude at the antipode of the sub-solar point, MLT is given by

$$MLT = \frac{(ML_{sat} - ML_{ant})}{15^\circ}$$

4. COORDINATE TRANSFORMATIONS

Many of the transformations of the coordinate systems given in chapter 3 are presented here. (Refer to chapters 2 and 3 for definitions of symbols and variables)

The transformation between geocentric coordinate systems is most easily accomplished through the use of a transformation matrix. If M is a transformation matrix that contains the rectangular positions of a new system's unit vectors in the original system, then the time-dependent transformation from the original rectangular system to the new rectangular system is

$$\overline{r_N} = M(N-O) * \overline{r_O}$$

where $\overline{r_O}$ and $\overline{r_N}$ are position vectors in rectangular old and new systems, respectively.

If the geocentric coordinate systems are not fixed with respect to one another, time dependency must be included such that

$$\overline{r_N} = M(t) * \overline{r_O}$$

where $M(t)$ is time dependent.

If a time-independent, geocentric transformation from the new system to the original system is desired,

$$\overline{r_O} = M^I * \overline{r_N}$$

where M^I is the inverse of matrix M.

4.1 SPHERICAL TO RECTANGULAR

In the Geocentric (GEOC), Geodetic (GEOD) and Geographic (GEOG) rectangular coordinate systems,

$$X_{GEOC} = X_{GEOD} = X_{GEOG}$$

$$Y_{GEOC} = Y_{GEOD} = Y_{GEOG}$$

$$Z_{GEOC} = Z_{GEOD} = Z_{GEOG}$$

4.1.1 Geocentric (GEOC) [*Riordan*, 1966]

The transformation of geocentric latitude, geocentric longitude, and the magnitude of the radius vector into rectangular coordinates is given by the following formulae:

$$X_{GEOC} = R * \cos(LAT_{GEOC}) * \cos(LON_{GEOC})$$

$$Y_{GEOC} = R * \cos(LAT_{GEOC}) * \sin(LON_{GEOC})$$

$$Z_{GEOC} = R * \sin(LAT_{GEOC})$$

4.1.2 Geodetic (GEOD) [*CIRRIS*, 1986]

The transformation of geodetic latitude, geodetic longitude, and the geodetic altitude into rectangular coordinates is given by the following formulae:

$$X_{GEOD} = (N+h) * \cos(LAT_{GEOD}) * \cos(LON_{GEOD})$$

$$Y_{GEOD} = (N+h) * \cos(LAT_{GEOD}) * \sin(LON_{GEOD})$$

$$Z_{GEOD} = [(1 - EC^2) * N + h] * \sin(LAT_{GEOD})$$

where EC = eccentricity of the reference ellipsoid, N = radius of curvature of the prime vertical given by

$$N = \frac{a}{\sqrt{(1 - EC^2 * \cos^2(LAT_{GEOD})}} ,$$

a = radius of the Earth, and h = geodetic altitude.

4.1.3 Geographic (GEOG)

In general, geographic latitude and longitude are considered equivalent to geodetic latitude and longitude. Thus, the formulae in section 4.1.2 above apply to the geographic system as well.

4.2 TRANSFORMATIONS TO EARTH CENTERED INERTIAL (ECI)

In the following discussion, only the transformation matrices are presented.

4.2.1 ECI* ← Geocentric (GEOC)

(* - Same as Geocentric Equatorial Inertial (GEI) and Geocentric Celestial Inertial (GCI))

$$M(ECI\text{-}GEOC) = \begin{vmatrix} \cos[GMST] & -\sin[GMST] & 0 \\ \sin[GMST] & \cos[GMST] & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

(Where GMST is the Greenwich Mean Sidereal Time angle (i. e., the angle between the Greenwich meridian and the first point of Aries measured eastward from the first point of Aries in the equatorial plane)

4.2.2 ECI ← Geodetic (GEOD)
ECI ← Geographic (GEOG)

Once the geodetic (or geographic) coordinates are expressed in equatorial rectangular coordinates, the transformation is identical to the geocentric to ECI transformation above. Thus,

$$M(ECI - GEOG) = M(ECI - GEOD) = M(ECI - GEOC)$$

4.3 TRANSFORMATIONS FROM EARTH CENTERED INERTIAL (ECI)

In the following discussion, only the transformation matrices are presented.

4.3.1 Geocentric (GEOC) ← ECI*

The Z-axis is colinear with Z_{ECI}, so

$$\overline{Z} = (0, 0, 1).$$

The X-axis points toward Greenwich,

$$\overline{X} = (\cos\alpha_G, \sin\alpha_G, 0)$$
$$\overline{Y} = \overline{Z} \times \overline{X}.$$

The origins are coincident so $d_{AB} = 0$.

4.3.2 Geocentric Solar Ecliptic (GSE) ← ECI* [*Russell*, 1971]

(GSE is the same as Ecliptic (EC) and Solar Ecliptic (SE))
(ECI is the same as Geocentric Equatorial Inertial (GEI) and Geocentric Celestial Inertial (GCI))

$$M(GSE-ECI) = \begin{vmatrix} \sin\theta\cos\theta & \sin\theta\sin\phi & \cos\theta \\ (-0.3976812*\cos\theta+0.9175237*\sin\theta\sin\phi) & (0.9175237*\sin\theta\cos\phi) & (0.3976812*\sin\theta\cos\phi) \\ 0 & -0.3976812 & 0.9175237 \end{vmatrix}$$

where θ is the declination and ϕ is the right ascension of the current solar position in ECI coordinates. (The Sun's spin axis is inclined 7.25° to the ecliptic.)

4.3.3 Geomagnetic (GEOM) ← ECI*

$$M(GEOM\text{-}ECI) = M(GEOM\text{-}GEOG) * M(GEOG\text{-}ECI) = M(GEOM\text{-}GEOG) * M(ECI\text{-}GEOG)^I$$

(see sections 4.4.2 and 4.2.2 for the above component matrices).

4.3.4 Geocentric Solar Magnetospheric (GSM) ← ECI*

The X-axis points toward the Sun and is colinear with a vector from the geomagnetic center d_{ECI} to the Sun s_{ECI} so

$$\overline{X} = \frac{(s_{ECI} - d_{ECI})}{|s_{ECI} - d_{ECI}|}.$$

The dipole axis is in the X-Z plane, so

$$\overline{Y} = \frac{(m_{\mathrm{ECI}} \times \overline{X})}{|m_{\mathrm{ECI}} \times \overline{X}|}$$

and

$$\overline{Z} = \overline{X} \times \overline{Y}$$

with

$$d_{\mathrm{AB}} = - d_{\mathrm{ECI}}.$$

4.3.5 Solar Magnetic (SM) ← ECI*

The Z-axis is along the dipole axis

$$\overline{Z} = \frac{m_{\mathrm{ECI}}}{|m_{\mathrm{ECI}}|} .$$

The origin is shifted by d_{ECI} and the Sun is in the X-Z plane.

$$\overline{Y} = \frac{(\overline{Z} \times (s_{\mathrm{ECI}} - d_{\mathrm{ECI}}))}{|\overline{Y}|}$$

and

$$\overline{X} = \overline{Y} \times \overline{Z}$$

with

$$d_{\mathrm{AB}} = - d_{\mathrm{ECI}}.$$

4.3.6 Vehicle-Dipole-Horizon (VDH) ← ECI*

The Z-axis is along the dipole,

$$\overline{Z} = \frac{m_{ECI}}{|m_{ECI}|} .$$

The vehicle is in the X-Z plane which passes through the geomagnetic center, d_{ECI}, thus,

$$\overline{Y} = \overline{Z} \times (r_{ECI} - d_{ECI})$$

and

$$\overline{X} = \overline{Y} \times \overline{Z} .$$

The origin is taken at the vehicle, so that

$$d_{AB} = r_{ECI}$$

but this system is rarely used for displacements.

4.3.7 Local Vertical Local Horizontal (LVLH) ← ECI*

The Z-axis is colinear with the vector from the vehicle to the geographic center of the Earth, positive inward.

$$\overline{Z} = \frac{-r_{ECI}}{|r_{ECI}|} .$$

$\overline{Z}$ and $\overline{X}$ form the instantaneous orbit plane of the vehicle, so that

$$\overline{Y} = \frac{\overline{Z} \times \overline{V}_{ECI}}{|\overline{Z} \times \overline{V}_{ECI}|}$$

and

$$\overline{X} = \overline{Y} \times \overline{Z}$$

with

$$d_{AB} = r_{ECI} .$$

4.4 OTHER GEOMAGNETIC TRANSFORMATIONS

In the following discussion, only the transformation matrices are present.

4.4.1 Solar Magnetic (SM) ↔ Geocentric Solar Magnetospheric (GSM) [*Russell*, 1971]

(SM is the same as Solar Geomagnetic (SG))
(GSM is the same as Solar Magnetospheric (SMC))

$$M(SM-GSM) = \begin{vmatrix} \cos\mu & 0 & -\sin\mu \\ 0 & 1 & 0 \\ \sin\mu & 0 & \cos\mu \end{vmatrix}$$

where μ is the dipole tilt angle.

4.4.2 Geomagnetic (GEOM) ↔ Geographic (GEOG) [*Russell*, 1971] or Dipole (D) ↔ Geographic (GEOG)

The northern geomagnetic pole is at geographic latitude 79.19° north (colatitude of 10.81°) and geographic longitude of 70.98° west (Epoch 1990).

$$M(GEOM - GEOG) = \begin{vmatrix} 0.320158 & -0.928599 & -0.187626 \\ 0.945388 & 0.325947 & 0.0 \\ 0.061156 & -0.177380 & 0.982240 \end{vmatrix}$$

[M(GEOM-GEOG) represents the geomagnetic X, Y, Z positions of the rectangular axes (of unit length) in the geographic X, Y, Z system. Row 1 corresponds to the X, Y, Z position of the geomagnetic X-axis of unit length in the geographic system. Row 2 applies to the geomagnetic Y-axis and row 3 applies to the geomagnetic Z-axis. Column 1 represents the position of the appropriate geomagnetic axis with respect to the geographic X-axis. Column 2 is with respect to the geographic Y-axis, and column 3 is with respect to the geographic Z-axis.]

$$M(GEOM - GEOG) * \overline{r_{GEOG}} = \overline{r_{GEOM}}$$

$$M(GEOG - GEOM) = M(GEOM - GEOG)^I$$

$$M(GEOG - GEOM) * \overline{r_{GEOM}} = \overline{r_{GEOG}}$$

4.4.3 Geomagnetic (GEOM) ← Geocentric (GEOC)

The Z-axis is colinear with the dipole axis which is assumed to intersect the geocentric center of the Earth.

$$\overline{Z} = \frac{m_{GEO}}{|m_{GEO}|} .$$

m_{GEO} can easily be found from m_{ECI} by the transformation of paragraph 4.3.4 (GEOC ← ECI). The south geographic pole, $\overline{g} = (0,0,-1)$ lies in the XZ geomagnetic plane and the origins are coincident, so

$$\overline{Y} = \overline{Z} \times \overline{g}$$

and

$$\overline{X} = \overline{Y} \times \overline{Z} .$$

The (GEOM ← ECI) conversion is easily carried out by doing 4.3.3.

4.5 ATTITUDE TRANSFORMATIONS AND QUATERNIONS

Attitude determination is concerned exclusively with establishing the angular or rotational orientation of an object - ususally the observer - with respect to some reference orthogonal triad, such as the Local Vertical Local Horizontal (LVLH) or the Body Axis (BA) coordinate systems. If we assume the yaw (y), pitch (p) and roll (r) sequence, the transformation that gives the new orientation of a vector in the original reference frame is given by the following matrix (S = sine, C = cosine):

$$M(N-O) = \begin{vmatrix} C_y C_p & -S_y C_r + C_y S_p S_r & S_y S_r + C_y S_p C_r \\ S_y C_p & C_y C_r + S_y S_p S_r & -C_y S_r + S_y S_p C_r \\ -S_p & C_p S_r & C_p C_r \end{vmatrix}$$

Alternatively, if the observer's orientation is the new one following the same yaw, pitch and roll sequence, the orientation of the original vector in the new reference frame is given by the following matrix transformation:

$$M(O-N) = \begin{vmatrix} C_y C_p & S_y C_p & -S_p \\ -S_y C_r + C_y S_p S_r & C_y C_r + S_y S_p S_r & C_p S_r \\ S_y S_r + C_y S_p C_r & -C_y S_r + S_y S_p C_r & C_p C_r \end{vmatrix}$$

It may be noted that $M(O-N)$ is the transpose, or inverse, of $M(N-O)$

$$M(O-N) = M^{T}(N-O) = M^{I}(N-O).$$

Attitude transformations of the type described above involve three rotation angles, called Euler angles, e.g. y, p and r. The rotations take place about the axes specified, each one in the intermediate reference frame determined by the previous rotation. The rotations are non-commutative, i.e., the angles to achieve a final orientation depend upon the sequence of axes specified. In fact, the three Euler angles may be specified about two axes only, such as in the x-axis, y-axis, x-axis sequence. This flexibility results in twelve possible representations for the attitude matrix [*Wertz*, 1986].

A fundamental property of attitude transformations relates to the existence of a <u>Euler Axis and Angle</u> which allows any desired orientation to be achieved about one fixed axis by a single angular rotation. If the 3x3 rotation matrix m is comprised of elements M_{ij} such that

$$M = \begin{vmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{vmatrix}$$

the rotation angle α is given by

$$\cos\alpha = \frac{1}{2}(M_{11} + M_{22} + M_{33} - 1)$$

and the single Euler axis unit vector $\overline{E}$ is given by

$$E_x = \frac{(M_{32} - M_{23})}{(2\sin\alpha)}$$

$$E_y = \frac{(M_{13} - M_{32})}{(2\sin\alpha)}$$

$$E_z = \frac{(M_{21} - M_{12})}{(2\sin\alpha)}$$

Inversely, the rotation matrix M is given by

$$M = \begin{vmatrix} \cos\alpha + E_x^2(1-\cos\alpha) & E_xE_y(1-\cos\alpha) + E_z\sin\alpha & E_xE_z(1-\cos\alpha) - E_y\sin\alpha \\ E_xE_y(1-\cos\alpha) - E_z\sin\alpha & \cos\alpha + E_y^2(1-\cos\alpha) & E_yE_z(1-\cos\alpha) + E_x\sin\alpha \\ E_xE_z(1-\cos\alpha) + E_y\sin\alpha & E_yE_z(1-\cos\alpha) - E_x\sin\alpha & \cos\alpha + E_z^2(1-\cos\alpha) \end{vmatrix}$$

4.5.1 Quaternions

A four parameter entity was introduced by Hamilton in the nineteenth century to represent rigid body rotations and consists of scalar and vector components as follows:

$$q = q_0 + iq_1 + jq_2 + kq_3$$

where i, j, k are the hyperimaginary numbers satisfying the conditions:

$$i^2 = j^2 = k^2 = -1$$

$$ij = -ji = k$$

$$jk = -kj = i$$

$$ki = -ik = j$$

and the conjugate or inverse of q is defined as:

$$q^* = q_0 - iq_1 - jq_2 - kq_3$$

The four parameters are also called the Euler symmetric parameters and are related to the Euler Axis and Angle components by

$$\begin{aligned}
q_0 &= \cos\alpha/2 \\
q_1 &= E_x \sin\alpha/2 = (M_{32} - M_{23})/4q_0 \\
q_2 &= E_y \sin\alpha/2 = (M_{13} - M_{31})/4q_0 \\
q_3 &= E_z \sin\alpha/2 = (M_{21} - M_{12})/4q_0
\end{aligned}$$

where $q_0^2 + q_1^2 + q_2^2 + q_3^2 = 1$.

NASA provides orbiter attitude data in terms of quaternions [*CIRRIS*, 1986; ***Cooper, et. al.***, 1985; ***Kendra***, 1990], and the following additional relationships may be derived:

If $\overline{r_O}$ and $\overline{r_N}$ are the vectors before and after rotation, respectively

$$\overline{r_N} = q\ \overline{r_O}\ q^*.$$

This is equivalent to $\overline{r_N} = M * \overline{r_O}$, where

$$M = \begin{vmatrix} q_0^2 + q_1^2 - q_2^2 - q_3^2 & 2(q_1 q_2 - q_0 q_3) & 2(q_0 q_2 + q_1 q_3) \\ 2(q_0 q_3 + q_1 q_2) & q_0^2 - q_1^2 + q_2^2 - q_3^2 & 2(q_2 q_3 - q_0 q_1) \\ 2(q_1 q_3 - q_0 q_2) & 2(q_0 q_1 + q_2 q_3) & q_0^2 - q_1^2 - q_2^2 + q_3^2 \end{vmatrix}$$

Inversely,

$$\begin{aligned}
M_{11} + M_{22} + M_{33} + 1 &= 4q_0^2 \\
M_{11} - M_{22} - M_{33} + 1 &= 4q_1^2 \\
-M_{11} + M_{22} - M_{33} + 1 &= 4q_2^2 \\
-M_{11} - M_{22} + M_{33} + 1 &= 4q_3^2
\end{aligned}$$

As a final note, extreme caution is required when using rotation operators. The final reference frame desired and the rotation sequence must be carefully specified, and the correct matrix representation differs in each case. Single and even compound 90° rotation tests may not diagnose a matrix error if the elements have been transposed. Even the quaternion rotation operation can be confused, since the definition $\bar{r}_N = \bar{q}^* \bar{r}_O \bar{q}$ may be used [*Wertz*, 1986], in which case the negative of the vector components $\bar{E}_x$, $\bar{E}_y$, $\bar{E}_z$ should be employed.

5. GEOMAGNETIC DIPOLE MODELS

Particles in space up to a few Earth radii are controlled by Earth's internal magnetic field. Further out, the influence of external factors, such as the ring currents and the solar wind, increases. Particle distributions and dynamics are accordingly best described using geomagnetically oriented coordinates. Key considerations that characterize these coordinate systems are

1) The tilt of the magnetic field dipole axis relative to Earth's rotational axis
2) The apparent offset of the magnetic center from Earth's center
3) Use of a solar reference direction at greater distances into the magnetosphere.

The description here concentrates on internal magnetic field considerations.

Models of the internal field follow the formulation of Gauss in which V, the magnetic field potential at any point in space, is expressed in a spherical harmonic expansion:

$$V = a\sum_{n=1}^{N}\left(\frac{a}{r}\right)^{n+1}\sum_{m=0}^{n}(g_n^m \cos m\phi + h_n^m \sin m\phi)P_n^m(\cos\theta)$$

where

$$a = \textit{mean Earth radius},\ \phi = \textit{longitude},\ \theta = \textit{colatitude}$$

and

$P_n^m(\sin\lambda)$ = *associated Legendre polynomial, where the normalized form of this function, called the Schmidt function, is now almost exclusively used.*

Updated models are provided for different epochs, typically for g_n^m and h_n^m coefficients through order n=m=10. Table 5-1 lists the Schmidt coefficients and their secular change for the IGRF85 model for epoch 1985.0.

The vector field is given by $\overline{B} = -\nabla V$ and the northward, eastward and downward components of the field are thus

Table 5-1. Schmidt Coefficients and Their Secular Change for IGRF 1985, Epoch 1985.0

		Main Field (gammas)		Secular Change (gammas/year)	
n	m	g_n^m	h_n^m	$\dot{g}_n^m$	$\dot{h}_n^m$
1	0	-29877.0	-	23.2	-
1	1	-1903.0	5497.0	10.0	-24.5
2	0	-2073.0	-	-13.7	-
2	1	3045.0	-2191.0	3.4	-11.5
2	2	1691.0	-309.0	7.0	-20.2
3	0	1300.0	-	5.1	-
3	1	-2208.0	-312.0	-4.6	5.3
3	2	1244.0	284.0	-0.6	2.3
3	3	835.0	-296.0	0.1	-10.8
4	0	937.0	-	0.1	-
4	1	780.0	233.0	-0.6	3.8
4	2	363.0	-250.0	-7.8	2.2
4	3	-426.0	68.0	-1.4	2.5
4	4	169.0	-298.0	-6.8	0.9
5	0	-215.0	-	1.3	-
5	1	356.0	47.0	0.1	0.1
5	2	253.0	148.0	-1.5	-0.2
5	3	-94.0	-155.0	-3.2	-0.1
5	4	-161.0	-75.0	0.1	0.6
5	5	-48.0	95.0	-0.1	0.0
6	0	52.0	-	1.4	-
6	1	65.0	-16.0	-0.3	-0.4
6	2	50.0	90.0	1.7	-1.1
6	3	-186.0	69.0	0.6	-0.8
6	4	4.0	-50.0	0.0	-2.3
6	5	17.0	-4.0	0.9	-0.5
6	6	-102.0	20.0	1.2	-0.1
7	0	75.0	-	0.2	-
7	1	-61.0	-82.0	-0.6	0.2
7	2	2.0	-26.0	-0.5	1.0
7	3	24.0	-1.0	0.8	1.1
7	4	-6.0	23.0	1.0	1.9
7	5	4.0	17.0	0.4	0.3
7	6	9.0	-21.0	-0.5	0.2
7	7	0.0	-6.0	-0.1	0.9
8	0	21.0	-	0.7	-
8	1	6.0	7.0	0.0	0.1
8	2	0.0	-21.0	0.3	-1.0
8	3	-11.0	5.0	0.4	0.1
8	4	-9.0	-25.0	-0.3	-0.8
8	5	2.0	11.0	-0.3	0.2
8	6	4.0	12.0	0.1	-0.8
8	7	4.0	-16.0	-0.5	-0.1
8	8	-6.0	-10.0	-0.8	1.3
9	0	5.0	-		
9	1	10.0	-21.0		
9	2	1.0	16.0		
9	3	-12.0	9.0		
9	4	9.0	-5.0		
9	5	-3.0	-6.0		
9	6	-1.0	9.0		
9	7	7.0	10.0		
9	8	2.0	-6.0		
9	9	-5.0	2.0		
10	0	-4.0	-		
10	1	-4.0	1.0		
10	2	2.0	0.0		
10	3	-5.0	3.0		
10	4	-2.0	6.0		
10	5	5.0	-4.0		
10	6	3.0	0.0		
10	7	1.0	-1.0		
10	8	2.0	4.0		
10	9	3.0	0.0		
10	10	0.0	-6.0		

$$X = \left(\frac{1}{r}\right)\left(\frac{\partial V}{\partial \theta}\right)$$

$$Y = -\left(\frac{1}{r\sin\theta}\right)\left(\frac{\partial V}{\partial \theta}\right)$$

$$Z = \frac{\partial V}{\partial r}$$

Besides being tilted to the rotational axis, Earth's magnetic field turns out to be both asymmetric as well as eccentric, i.e. the North and South magnetic poles where the field is downward exclusively (dip poles) are not located at diametrically opposed geographic latitudes or longitudes. This and the higher order spherical variations in the terrestrial magnetic field are implicit in the values assigned to the Schmidt coefficients.

High speed computers coupled with recursion algorithms allow complete calculation of magnetic field characteristics nowadays, but some simple models are regularly cited, since they offer good intuition and may even be adequate for modeling purposes. Chapman and Bartels [1940] and Fraser-Smith [1987] address this problem, and three basic models for which geographic/geomagnetic coordinate transformations may be defined are discussed below:

1) Centered dipole model
2) Eccentric dipole model
3) Dip pole model.

The centered dipole model is determined analytically by the first order Schmidt coefficients exclusively, *viz.* g_1^0, g_1^1, h_1^1. The eccentric dipole model is based on the first and second order Schmidt coefficients, so that five additional constants are involved, *viz.* $g_2^0, g_2^1, h_2^1, g_2^2, h_2^2$, and an analytical formulation is again possible. However, these first two models fail to locate the dip poles by a few hundred kilometers on Earth's surface, and an alternate model may be determined from the complete magnetic field model.

5.1 CENTERED DIPOLE MODEL

The first order geomagnetic potential is:

$$V_1 = a\left(\frac{a}{r}\right)^2\left[g_1^0\cos\theta + (g_1^1\cos\phi + h_1^1\sin\phi)\sin\theta\right]$$

while the simple dipole model is

$$V_1 = a\left(\frac{a}{r}\right)^2 H_0\cos W$$

where W is the dipole colatitude.

Thus,

$$H_0^2 = g_1^0 g_1^0 + g_1^1 g_1^1 + h_1^1 h_1^1$$

and

$$\cos W = \cos\theta \cos\theta_0 + \sin\theta \sin\theta_0 \cos(\phi - \phi_0)$$

where

$$\cos\theta_0 - \frac{g_1^0}{H_0}, \qquad \tan\phi_0 = \frac{h_1^1}{g_1^1}.$$

This shows that the simple dipole model has moment H_0, and its dipole axis is oriented in geographic coordinates according to:

$$(g_1^1, h_1^1, g_1^0).$$

θ_0, ϕ_0 are seen to be the colatitude and longitude, respectively, of the dipole pole.

5.2 ECCENTRIC DIPOLE MODEL

The second order geomagnetic potential can also be represented by a simple dipole model, except for the terms corresponding to second harmonics of the longitude [*Chapman and Bartels*, 1940]. The representation turns out to be identical to the dipole model defined in the previous section, but the dipole is eccentric as follows:

If we define

$$L_0 = 2g_1^2 g_2^0 + (g_1^1 g_2^1 + h_1^1 h_2^1)\sqrt{3}$$

$$L_1 = -g_1^1 g_2^0 + (g_1^0 g_2^1 + g_1^1 g_2^2 + h_1^1 h_2^2)\sqrt{3}$$

$$L = -h_1^1 g_2^0 + (g_1^0 h_2^1 - h_1^1 g_2^2 + g_1^1 h_2^2)\sqrt{3}$$

$$E = (L_0 g_1^0 + L_1 g_1^1 + L_2 h_1^1)/4H_0^2$$

the center of the eccentric dipole is given by

$$x_0 = a(L_1 - g_1^1 E)/3H_0^2$$

$$y_0 = a(L_2 - h_1^1 E)/3H_0^2$$

$$z_0 = a(L_0 - g_1^0 E)/3H_0^2.$$

The dipole moment $H_0 a^3$ and axis inclination (g_1^1, h_1^1, g_1^0) remain unchanged. It should be noted that this axis is only approximately perpendicular to the radius vector (x_0, y_0, z_0).

5.3 DIP POLE MODEL

Another model could be considered if one is primarily interested in magnetic fields at the Earth's surface, and in particular, the dip pole regions where the magnetic field is vertical to the surface. The location of these dip poles can be readily found from a computational search, using the full scale geomagnetic model. The choice of eccentric dipole parameters to model this case is not straightforward, since the axis of the dipole that passes through the dip poles is not necessarily perpendicular to the Earth's surface at those points. Fraser-Smith [1987] shows graphically how the centered dipole poles, the eccentric dipole poles and the eccentric dipole dip poles fan out progressively, with the actual dip poles being even further away, especially for the south polar region. Parameters for the three dipole models have been determined using IGRF 1985 for Epoch 1990.0, and are presented in Table 5-2.

Table 5-2. Internal Magnetic Field Dipole Models for Epoch 1990.0

Internal Magnetic Field Dipole Models for Epoch 1990.0	Centered Dipole	Eccentric Dipole	Dip Pole
North Pole Lat, Long	79.19°N, 289.02°E	82.4°, 258.8°E	77.8°N, 256.3°E
South Pole Lat, Long	79.19°S, 109.02°E	75.2°S, 119.3°E	64.9°S, 139.4°E
Dipole Center (X, Y, Z, Km)	0, 0, 0	-398.25, 267.25, 187.06	--
Dipole Axis Orientation (θ, ϕ deg)	79.19°, 289.02°	79.19°, 289.02°	
Earth Center to Dipole Axis distance (Km)	0	~500	~1200
Pole Migration (Km/yr)	~1.0	~1.5	~5.0

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