**Definition of quaternion in green book and ADM**:

Green book:

The attitude of the body frame with respect to the reference frame is represented by a unique rotation around a vector **u**, which is invariant in both frames, with an angular amplitude Φ. The vector **u** is oriented in such a way that makes Φ positive directly around the **u** vector in the movement from reference frame to body frame.

At this rotation is associated a unit quaternion Q = {cos(Φ/2),**u** sin(Φ/2)}. The scalar component of this 4-vector, cos(Φ/2) ≡ QC, is conventionally written as either the first or last component. Care must be taken to ensure that the same convention is used by exchange participants. In the following description the convention placing the scalar first is used.

The attitude quaternion is defined by a 4-dimension vector Q (QC,Q1,Q2,Q3) with:

* QC = cos(Φ/2);
* Q1 = e1\*sin(Φ/2);
* Q2 = e2\*sin(Φ/2);
* Q3 = e3\*sin(Φ/2).

Where Φ is the rotation angle between the reference frame and the body frame and e1, e2, and e3 are the components of the unit rotation vector **u** in the body axis (or in the reference frame) with the relation

QC\*QC + Q1\*Q1 + Q2\*Q2 + Q3\*Q3 = 1

Also defined is the conjugate quaternion Q\* =( QC, -Q1, -Q2, Q3

With

 xi the components of a vector in the reference frame, with **xi**= (xi1,xi2,xi3)

 xf the components of a vector in the body frame, with **xf** = (xf1,xf2,xf3)

xf and xi are linked by

xf = Q \* xi \* QT and xi = QT \* xf\* Q.

(These products are defined by the quaternion algebra.)

This link can also be expressed using a rotation matrix *M*.





ADM

The quaternion called "from frame A to frame B" is defined as the quaternion of the rotation that transforms the basis vectors of frame A into the basis vectors of frame B. That is to say that the basis vectors of frame B are the respective images of the basis vectors of frame A by the rotation.

The quaternion is defined by four components:

 q1 = sin(/2) \* e1

q2 = sin(/2) \* e2

q3 = sin(/2) \* e3

 qc = cos(/2)

Where:

is the rotation angle,

e1, e2 and e3 are the coordinates of the rotation axis in either frame A or frame B.

The quaternion is related to the frame transformation matrix in the following way:

Let XA be the coordinates of some vector in frame A, and XB the coordinates of the same vector in frame B.

The frame transformation matrix MBA that transforms coordinates in frame A to coordinates in frame B is defined by:

 XB = MBA \* XA

where MBA is a function of the quaternion compinents:

$$M\_{BA}=\left⌈\begin{matrix}q\_{1}^{2}-q\_{2}^{2}-q\_{3}^{2}+q\_{c}^{2}&2 (q\_{1} q\_{2}+q\_{3} q\_{c})&2 (q\_{1} q\_{3}-q\_{2} q\_{c})\\2 (q\_{1} q\_{2}-q\_{3} q\_{c})&-q\_{1}^{2}+q\_{2}^{2}-q\_{3}^{2}+q\_{c}^{2}&2 (q\_{2} q\_{3}+q\_{1} q\_{c})\\2 (q\_{1} q\_{3}+q\_{2} q\_{c})&2 (q\_{2} q\_{3}-q\_{1} q\_{c})&-q\_{1}^{2}-q\_{2}^{2}+q\_{3}^{2}+q\_{c}^{2}\end{matrix}\right⌉$$

Conclusion

The frame transformation matrices are the same (in particular the non diagonal terms).

The matrices define the coordinate transformation from ref frame to body frame in green book and from frame A to frame B in the ADM (where by definition the rotation transforms the basis vectors of frame A into the basis vectors of frame B).

So the definitions / conventions in the green book and in the ADM appear to be the same.