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A new approach has been taken to developing an ephemeris compression model. The model uses the two body and J_2 short-period periodic solution as a basis and overlays Fourier series to model the small differences between this analytic solution and the more accurate numerical reference. The ephemeris compression model runs faster than currently used GP models, accurately imitates the SP reference within a few hundred meters for several days, is valid for all eccentricities and inclinations, and requires transmission of less than 40 coefficients.

INTRODUCTION

The catalog of man-made objects in orbit about the Earth currently numbers nearly 10,000. Although this catalog is currently maintained using general perturbations (GP) orbit models, there are several current efforts aimed at maintaining a significant portion of the catalog to a higher accuracy using special perturbations (SP) orbit models. Such experiments are being made possible by the unprecedented growth of computational power available today.

Both the GP and the SP methods produce an element set or state vector, respectively, referenced to a specific epoch. The user of the element set or state vector must have a means of propagating the epoch information to other times of interest. Currently, users of GP element sets employ GP models such as SGP, SGP4 or PPT for their predictions. On the other hand, current users of SP vectors must be careful to adopt for their use exactly the same SP model as was employed for development of the vectors originally. These users tend to be very specialized and are interested in only a handful of satellites so the burden of SP runtime and the difficulty of implementing an SP model are acceptable to them.

In order for SP catalog experiments to be totally successful, it must be demonstrated that the products can be and are used by more than the small set of users. Otherwise, it becomes just an interesting experiment with no practical reason having been demonstrated for SP maintenance of more than the current small number of specialized satellites. To develop a collection of users, the data product should be

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easy to use, provide a clear improvement over GP prediction accuracy, and not have a great impact on computer resources.

Ephemeris compression is one possible option for providing a user with a fast running model which provides a good imitation of the SP accuracy. Ephemeris compression is a means of achieving a compact representation which approximates a time series of satellite positions. Typically the time series of satellite positions is first generated with a precise numerical integration of the equations of motion. Such methods are commonly used for representing planetary ephemerides[1]. Some research has been done to apply ephemeris compression to satellite orbits. Representative among these is the paper by Deprit[2]. More recently a paper by Coffey[3] provides the most extensive examination to date of the application of ephemeris compression to a large catalog of Earth satellites. Most of the previous approaches used a single set of basis functions for the ephemeris compression. They generally calibrated the coefficients of these functions by directly fitting inertial space coordinate data. As a result, the applicability was limited to only those satellites with small eccentricity. Most of these approaches required hundreds of terms to achieve a good imitation of the SP theory for an extended period of time. A radically different approach was suggested by Hoots[4]. This work demonstrated the use of a combined power series in time and a Fourier series to obtain good quality ephemeris compression over several day periods with less than 40 coefficients.

HYBRID EPHEMERIS COMPRESSION MODEL

The rectangular coordinates of a satellite ephemeris experience large variations each revolution primarily due to the main two body motion of the satellite. Any attempt to model this variation with a set of basis functions will require a great number of terms just to model the two body motion. Then additional terms will be required to model the perturbations superimposed on the two body motion. A new approach has been taken to developing an ephemeris compression model which builds around the known physics of the problem. This approach has an analogy to the semianalytic orbit model approach. Just as the semianalytic method adopts an analytic solution for the main problem and treats the remainder numerically, this new ephemeris compression model adopts an analytic solution for the two body and J_2 short-period portion of the problem and treats the remainder of the problem with a numerically developed Fourier series.

This method of ephemeris compression can be considered a hybrid method. It provides a unique marriage of GP and SP methods. The functional form of GP theories provides insight into the optimum selection of basis functions for the ephemeris compression model while the SP theory provides the reference orbit which is imitated by the ephemeris compression model. Thus, the model is known as a Hybrid Ephemeris Compression Model (HECM).

There are many solutions which could have been adopted for the analytic portion of the model. The objective here was to strike a balance between simplicity and accurate modeling. If we consider the two body portion of the problem to be zeroth order and the J_2 perturbations to be first order, then all other perturbations due to geopotential, third body and atmospheric drag are generally of size second order. Although we could include solutions which contain some of the second order effects, most would still not be included. It is simpler to include the two body and J_2 effects and let the Fourier series model all second order effects.

Because the model is based on the two body and J_2 solution, the power and flexibility of the Fourier approach can focus on modeling only the small differences between the analytic solution and the more accurate numerical integration. Thus, as a state vector is being created at the central site, it can be used to generate a trajectory file. This reference file can be used to calibrate the HECM to best imitate the reference file. Then the coefficients of the HECM can be forwarded to users who then use the HECM for their propagation.

MODEL DEVELOPMENT

In addition to the two body and J_2 short-period periodics, the model must also be able to successfully model the secular motion of the satellite. Based on the solution of Brouwer[5] as well as other works[6] which have added drag to the Brouwer solution, a time series is chosen to model the total secular motion of the satellite due to atmospheric drag and gravitational effects. Let the secular values of the orbital elements at time t be denoted by double-primed variables. Then the model is

$$\begin{aligned}
n'' &= n_o + n_1 t + n_2 t^2 + n_3 t^3 \\
e'' &= e_o + e_1 t + e_2 t^2 \\
i'' &= i_o + i_1 t \\
\Omega'' &= \Omega_o + \Omega_1 t + \Omega_2 t^2 \\
\omega'' &= \omega_o + \omega_1 t + \omega_2 t^2 \\
M'' &= M_o + M_1 t + \tfrac{1}{2} n_1 t^2 + 1/3\, n_2 t^3 + \tfrac{1}{4} n_3 t^4
\end{aligned} \qquad (1)$$

where

n = mean motion
e = eccentricity
i = inclination
Ω = right ascension of ascending node
ω = argument of perigee
M = mean anomaly
t = time since epoch and subscript o denotes a value at epoch time

and the subscripted parameters are solved for as a best fit to the reference numerical ephemeris. Next, we must add the J_2 short-period periodics. A computationally efficient formulation has been provided by Hoots[7]. The solution is a reformulation of the Brouwer solution in terms of variables which are nonsingular in eccentricity and inclination. Additionally, the method avoids a second solution of Kepler's equation through the choice of variables. Finally, since we only adopt the J_2 short-period periodic portion of the Brouwer solution, HECM has no singularity at the critical inclination. Introduce the variables

$$r = a\,\beta^2 / (1 + e \cos f)$$
$$y_4 = \sin i/2 \sin u$$
$$y_5 = \sin i/2 \cos u \qquad (2)$$
$$\lambda = f + \omega + \Omega$$

where a = semimajor axis, $\beta^2 = 1 - e^2$, f = true anomaly, and u = true argument of latitude.

After computing the secular effects at the time of interest, the double-primed variables are used in Kepler's equation to compute the double-primed true anomaly f". Then f" and the other double-primed variables can be used to compute r", y_4", y_5", and λ". The short period periodics are included through the equations

$$r' = r'' + \delta r$$
$$y_4' = y_4'' + \delta y_4 = \sin i''/2 \sin u'' + \cos u'' \sin i''/2\, \delta u + \tfrac{1}{2} \sin u'' \cos i''/2\, \delta i$$
$$y_5' = y_5'' + \delta y_5 = \sin i''/2 \cos u'' - \sin u'' \sin i''/2\, \delta u + \tfrac{1}{2} \cos u'' \cos i''/2\, \delta i$$
$$\lambda' = \lambda'' + \delta\lambda \qquad (3)$$

where

$$\delta r = a\beta^2 \{C_1 (-1 + 3\,\theta^2) [1 + 2r / a\beta + e \cos f / (1 + \beta)] - C_1 (1 - \theta^2) \cos 2u\}$$

$$\delta u = - C_1 (-1 + 3\,\theta^2) (1 - \beta) [e / (1 + \beta) + \cos f] \sin f$$

$$- \tfrac{1}{2} C_1 [(1 - 7\,\theta^2) \sin 2u + 2\, e\, (2 - 5\,\theta^2) \sin (f + 2\,\omega)$$

$$- 2\, e\, \theta^2 \sin (3f + 2\,\omega)] - 3\, C_1 (-1 + 5\,\theta^2) (f - M + e \sin f) \qquad (4)$$

$$\delta i = - C_1\, \theta \sin i\, [3 \cos 2u + 3\, e \cos (f + 2\omega) + e \cos (3f + 2\omega)]$$

$$\delta\lambda = C_1\, \theta\, [6 (f - M + e \sin f) - 3 \sin 2u - 3\, e \sin (f + 2\omega) - e \sin (3f + 2\omega)]$$

where $\theta = \cos i$, $C_1 = - \tfrac{1}{4} J_2 R^2 / a^2\beta^4$ and R = Earth equatorial radius with all variables on the right-hand side being double-primed variables.

Single-primed position is now calculated from

$$x' = r' [2 y_4' (y_5' \sin \lambda' - y_4' \cos \lambda') + \cos \lambda']$$
$$y' = r' [-2 y_4' (y_5' \cos \lambda' + y_4' \sin \lambda') + \sin \lambda'] \quad (5)$$
$$z' = r' [2 y_4' \cos i'/2]$$

and position has been calculated with only one solution of Kepler's equation.

The model contains 17 parameters which must be computed based on the reference ephemeris. A Gauss least squares is used to find values of the parameters which create a best fit of the secular and first-order short-period portion of the hybrid model to the reference ephemeris. This fit is done for the entire length of the reference ephemeris. Once the 17 parameters have been computed, most, if not all, of the secular and first-order short-period character of the reference ephemeris will have been captured in the HECM parameters. All that remains to be modeled are second-order periodic variations. Returning again to knowledge of the Brouwer solution, these variations are known to have period equal to the period of the satellite. A natural choice of basis functions to model this is trigonometric functions. The model chosen for the periodic variation is the first few terms of a Fourier series.

$$\Delta x = a_{xo} + \Sigma\, a_{xk} \cos (ku'') + \Sigma\, b_{xk} \sin (ku'')$$
$$\Delta y = a_{yo} + \Sigma\, a_{yk} \cos (ku'') + \Sigma\, b_{yk} \sin (ku'') \quad (6)$$
$$\Delta z = a_{zo} + \Sigma\, a_{zk} \cos (ku'') + \Sigma\, b_{zk} \sin (ku'')$$

where $k = 1$ to 3 in all sums.

The Fourier coefficients are obtained by first creating from the reference ephemeris a series of points equally spaced in true argument of latitude over the interval $[-\pi,\pi]$. It is only necessary to create this set of points for the first revolution of the reference ephemeris. The reason is that the Fourier series is assumed to be periodic with period equal to the satellite period. If the calibration of the 17 parameters successfully removed all secular effects, then the period associated with the predicted u" will changed secularly so that the 2π periodicity will hold true throughout the time span of the reference ephemeris. Let

$$u_1, u_2, u_3 \ldots u_q \quad (7)$$

be the set of q points equally spaced in true argument of latitude and covering the interval $[-\pi,\pi]$. The HECM can be used to provide a prediction of the single-primed position at each of these sample points. Let

$$\delta x(u_1), \delta x(u_2), \delta x(u_3) \ldots \delta x(u_q)$$
$$\delta y(u_1), \delta y(u_2), \delta y(u_3) \ldots \delta y(u_q) \quad (8)$$
$$\delta z(u_1), \delta z(u_2), \delta z(u_3) \ldots \delta z(u_q)$$

denote the differences between the reference ephemeris and the single-primed positions predicted with the HECM. The Fourier coefficients can be calculated from

$$
\begin{aligned}
a_{xo} &= 1/q\ [\ \delta x(u_1) + \delta x(u_2) + \ldots + \delta x(u_q)\ ] \\
a_{xk} &= 2/q\ [\ \delta x(u_1) \cos(ku_1) + \delta x(u_2) \cos(ku_2) + \ldots + \delta x(u_q) \cos(ku_q)] \\
b_{xk} &= 2/q\ [\ \delta x(u_1) \sin(ku_1) + \delta x(u_2) \sin(ku_2) + \ldots + \delta x(u_q) \sin(ku_q)] \\
\\
a_{yo} &= 1/q\ [\ \delta y(u_1) + \delta y(u_2) + \ldots + \delta y(u_q)\ ] \\
a_{yk} &= 2/q\ [\ \delta y(u_1) \cos(ku_1) + \delta y(u_2) \cos(ku_2) + \ldots + \delta y(u_q) \cos(ku_q)] \qquad (9) \\
b_{yk} &= 2/q\ [\ \delta y(u_1) \sin(ku_1) + \delta y(u_2) \sin(ku_2) + \ldots + \delta y(u_q) \sin(ku_q)] \\
\\
a_{zo} &= 1/q\ [\ \delta z(u_1) + \delta z(u_2) + \ldots + \delta z(u_q)\ ] \\
a_{zk} &= 2/q\ [\ \delta z(u_1) \cos(ku_1) + \delta z(u_2) \cos(ku_2) + \ldots + \delta z(u_q) \cos(ku_q)] \\
b_{zk} &= 2/q\ [\ \delta z(u_1) \sin(ku_1) + \delta z(u_2) \sin(ku_2) + \ldots + \delta z(u_q) \sin(ku_q)]
\end{aligned}
$$

Once the Fourier coefficients are computed, osculating position can be obtained from

$$
\begin{aligned}
x &= x' + \Delta x \\
y &= y' + \Delta y \qquad (10) \\
z &= z' + \Delta z
\end{aligned}
$$

Since k is chosen to be 3, there will be a total of 21 Fourier coefficients. Adding this to the 17 secular coefficients gives a total of 38 parameters needed for the HECM.

MODEL TESTING

In order to asses the capability of the HECM, a series of tests was performed. First, a set of over 70 element sets was selected to provide a reasonable sample over a wide range of eccentricities, inclinations, and mean motions. The set was also selected to span a range of perturbations from high drag through deep space. The characteristics of the sample element sets are given in the following Table.

Case #	Perigee (km)	Apogee (km)	Eccentricity	Inclination (deg)
1	959	10,791	0.40120	42.7
2	35,753	35,833	0.00096	24.9
3	1,171	38,411	0.71154	16.7
4	2,780	14,796	0.39614	32.2
5	5,825	5,864	0.00159	120.9
6	25,835	36,131	0.13779	16.3
7	2,378	5,277	0.14202	55.9
8	1,178	1,211	0.00219	74.0
9	294	521	0.01667	28.8

Case #	Perigee (km)	Apogee (km)	Eccentricity	Inclination (deg)
10	13,449	13,773	0.00810	124.8
11	5,839	5,948	0.00444	109.9
12	30,131	41,431	0.13402	35.4
13	776	14,107	0.48232	63.9
14	21,990	49,572	0.32712	23.5
15	8,185	35,619	0.48503	9.2
16	19,062	19,076	0.00028	65.0
17	36,004	36,055	0.00059	0.6
18	257	718	0.03356	28.1
19	35,748	35,826	0.00093	0.0
20	778	781	0.00025	108.1
21	1,047	1,091	0.00297	73.3
22	1,035	1,175	0.00935	89.8
23	20,111	20,254	0.00270	55.2
24	2,259	4,899	0.13254	86.9
25	35,752	36,330	0.00682	11.2
26	34,135	36,517	0.02856	13.9
27	35,834	36,933	0.01285	15.2
28	1,029	1,048	0.00133	99.4
29	912	40,204	0.72935	63.7
30	175	10,318	0.43629	34.6
31	450	1,263	0.05621	66.0
32	1,417	1,480	0.00407	74.0
33	1,400	1,415	0.00097	82.6
34	5,615	5,950	0.01380	52.6
35	2,308	38,043	0.67290	64.0
36	1,108	39,245	0.71809	63.8
37	1,664	38,150	0.69405	63.4
38	19,115	19,145	0.00057	65.0
39	297	5,266	0.27126	41.2
40	36,036	36,115	0.00093	25.8
41	216	6,777	0.33220	27.3
42	298	1,056	0.05372	36.2
43	290	1,488	0.08246	82.9
44	312	2,005	0.11233	23.1
45	35,937	36,089	0.00180	1.1
46	1,183	1,210	0.00176	82.6
47	2,949	37,097	0.64671	24.7
48	397	407	0.00074	82.5
49	1,077	39,278	0.71928	63.4

Case #	Perigee (km)	Apogee (km)	Eccentricity	Inclination (deg)
50	588	38,814	0.73290	63.4
51	537	1,992	0.09523	19.7
52	435	2,733	0.14433	82.6
53	1,480	1,524	0.00279	73.6
54	35,532	35,750	0.00259	2.0
55	1,882	2,163	0.01668	64.8
56	1,399	1,418	0.00119	82.6
57	1,413	1,471	0.00370	82.6
58	226	274	0.00361	64.9
59	35,774	35,798	0.00029	4.9
60	289	302	0.00098	51.6
61	526	562	0.00257	97.5
62	403	419	0.00118	65.0
63	649	687	0.00270	98.0
64	35,685	35,754	0.00081	2.1
65	196	455	0.01932	62.8
66	629	671	0.00296	82.5
67	237	285	0.00358	64.9
68	259	778	0.03762	82.9
69	35,721	35,873	0.00181	3.1
70	259	269	0.00072	39.0
71	390	396	0.00044	51.6
72	849	852	0.00021	71.0
73	649	649	0.00000	63.4

For each of the sample orbital element sets, a reference ephemeris was generated with points saved every one minute. The reference ephemeris was generated using a Gauss-Jackson 8^{th} order numerical integrator. The force model selected was a 12^{th} order geopotential, a Jacchia 1970 atmospheric density model, and a point mass representation for lunar and solar gravitational perturbations.

In each case the fit span selected for the HECM was the first 4 days of the reference ephemeris. The calibration of the 38 parameters for each case was accomplished as described earlier. A prediction using HECM was then performed and compared with the reference ephemeris for the 4 days which were fit. The quality of the comparison is described as the RMS of the differences between the HECM prediction and the reference ephemeris. The results for each case are presented in the following Table.

Case #	Perigee Height (km)	Apogee Height (km)	Fit RMS (m)
1	959	10,791	152
2	35,753	35,833	398
3	1,171	38,411	214
4	2,780	14,796	65
5	5,825	5,864	84
6	25,835	36,131	607
7	2,378	5,277	113
8	1,178	1,211	321
9	294	521	354
10	13,449	13,773	39
11	5,839	5,948	86
12	30,131	41,431	457
13	776	14,107	319
14	21,990	49,572	437
15	8,185	35,619	253
16	19,062	19,076	125
17	36,004	36,055	424
18	257	718	364
19	35,748	35,826	432
20	778	781	375
21	1,047	1,091	383
22	1,035	1,175	311
23	20,111	20,254	124
24	2,259	4,899	178
25	35,752	36,330	412
26	34,135	36,517	390
27	35,834	36,933	498
28	1,029	1,048	400
29	912	40,204	184
30	175	10,318	130
31	450	1,263	328
32	1,417	1,480	206
33	1,400	1,415	330
34	5,615	5,950	68
35	2,308	38,043	180
36	1,108	39,245	304
37	1,664	38,150	337
38	19,115	19,145	154
39	297	5,266	201

Case #	Perigee Height (km)	Apogee Height (km)	Fit RMS (m)
40	36,036	36,115	399
41	216	6,777	165
42	298	1,056	298
43	290	1,488	460
44	312	2,005	261
45	35,937	36,089	539
46	1,183	1,210	357
47	2,949	37,097	204
48	397	407	521
49	1,077	39,278	203
50	588	38,814	151
51	537	1,992	258
52	435	2,733	285
53	1,480	1,524	374
54	35,532	35,750	410
55	1,882	2,163	271
56	1,399	1,418	276
57	1,413	1,471	278
58	226	274	384
59	35,774	35,798	374
60	289	302	482
61	526	562	425
62	403	419	409
63	649	687	443
64	35,685	35,754	485
65	196	455	434
66	629	671	433
67	237	285	389
68	259	778	503
69	35,721	35,873	273
70	259	269	350
71	390	396	291
72	849	852	381
73	649	649	418

The results show that the HECM provides a good match to the reference ephemeris for a wide variety of eccentricities, inclinations, and mean motions. The RMS of the 4 day fit averages about 300 meters with a worst case of about 600 meters. The 38 parameters required to produce these results is an order of magnitude smaller than the number of parameters used in previously published methods while the results provide a closer fit to the reference. Additionally, unlike other ephemeris compression methods, this method applies to all eccentricities and includes all relevant perturbations.

Since the functional form of HECM is based on the general character of physical GP theories, the long term behavior of the HECM continues to approximate the true motion even when used at time points outside the fit interval. This is a sharp contrast to ephemeris compression methods using Chebychev polynomials which vary wildly beyond their normal interval. To asses this claim, each test case was also predicted for 4 days beyond the 4 day fit interval. The predictions were compared with the reference ephemeris and an RMS was computed. All cases had a graceful degradation with an RMS of tens of kilometers or less for the 4 days beyond the fit interval. Such a quality is extremely important for users in the event that a new set of ephemeris compression parameters is not received in a timely fashion.

CONCLUSIONS

Use of a combination of time polynomials and trigonometric series appears to be very effective for satellite ephemeris compression. The inclusion of the two body equations and J_2 short-period periodics as an integral part of the ephemeris compression model allows a significant economization of the number of terms required to achieve a given accuracy. It also removes any significant dependence on the eccentricity of the orbit. The methods is able to model all relevant perturbations of Earth orbiting satellites.

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