

Block Interleaved RS Codes for the Burst Erasure Channel

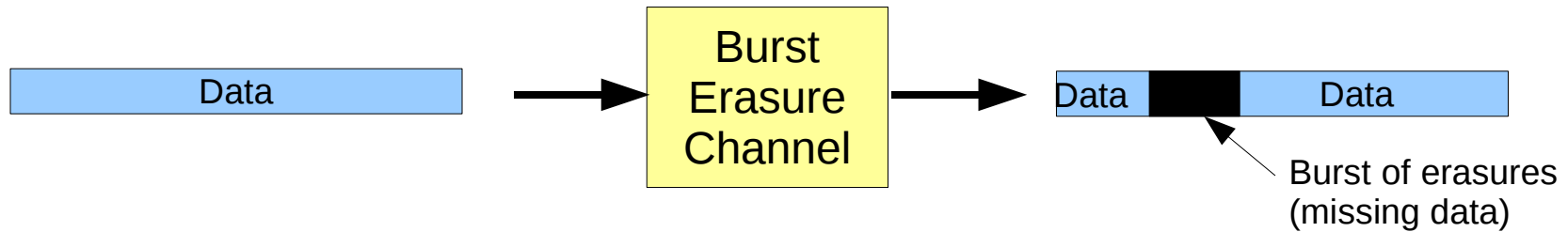
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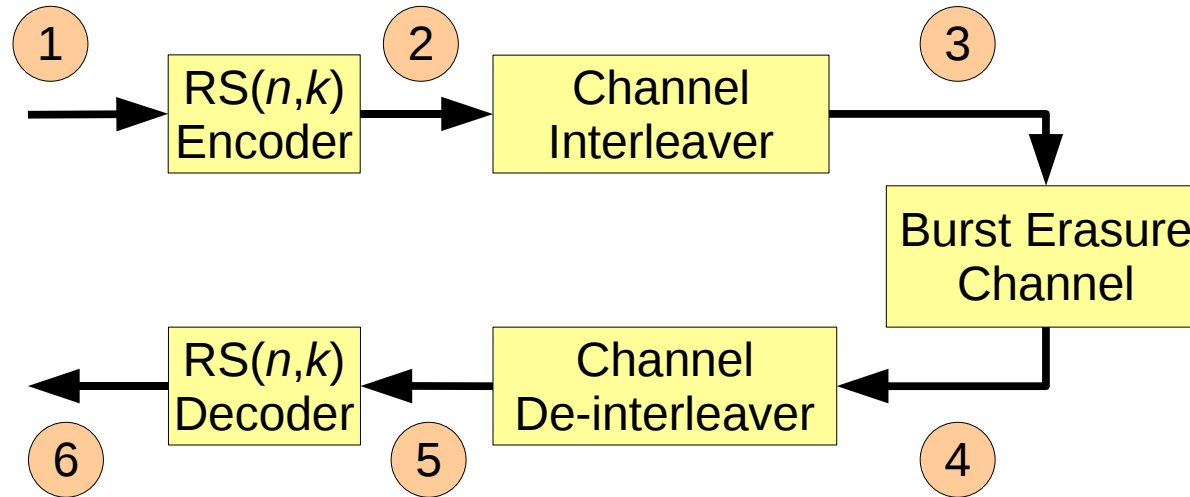
The Burst Erasure Correction Problem

The goal is to protect against a long burst of erasures:



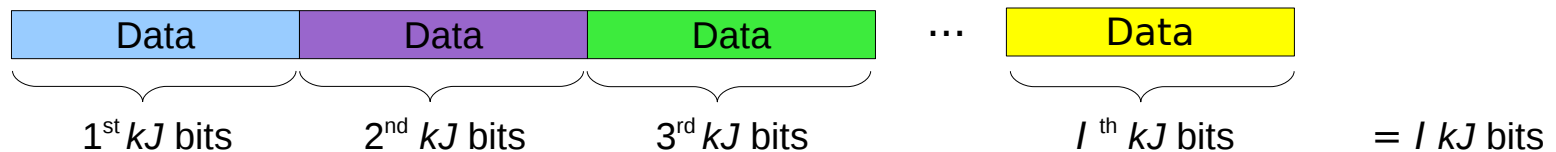
- Burst erasures occur on the higher layers, in the form of missing packets
- Metrics to compare codes by:
 - L_{\max} : Maximum length of guaranteed-correctable channel symbol erasure burst
 - By Singleton bound, $L_{\max} \leq n-k$ for (n,k) linear code
 - Define *efficiency* as $\eta = L_{\max}/(n-k)$ and *inefficiency* as $\mu = 1 - \eta$
 - Latency, complexity, bandwidth efficiency
- Questions to answer:
 - What kind of code should be used? What length? What rate?
 - Should an interleaver be used?
 - Short code with long interleaver?
 - Long code with short or no interleaver?

Interleaved RS Codes on Burst Erasure Channel



Data flow:

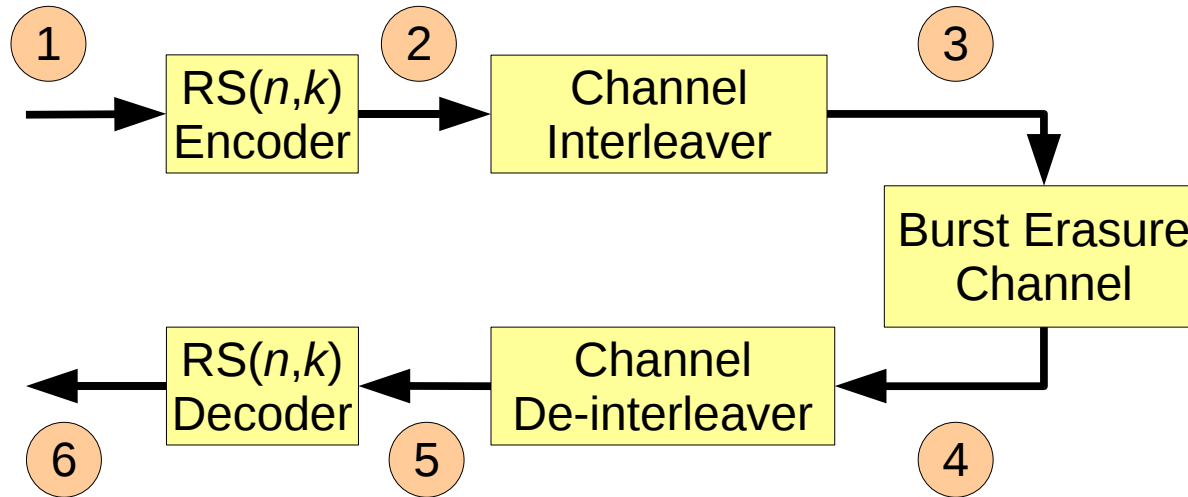
1 Information messages:



Each RS codeword has k input symbols

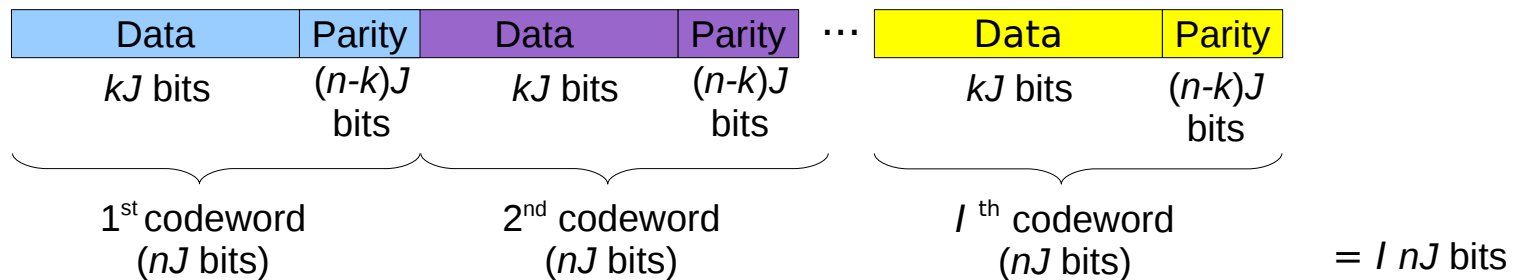
Each symbol has $J = \log_2(n+1)$ bits (for CCSDS RS(255,223) code, $J = 8$)

Interleaved RS Codes on Burst Erasure Channel

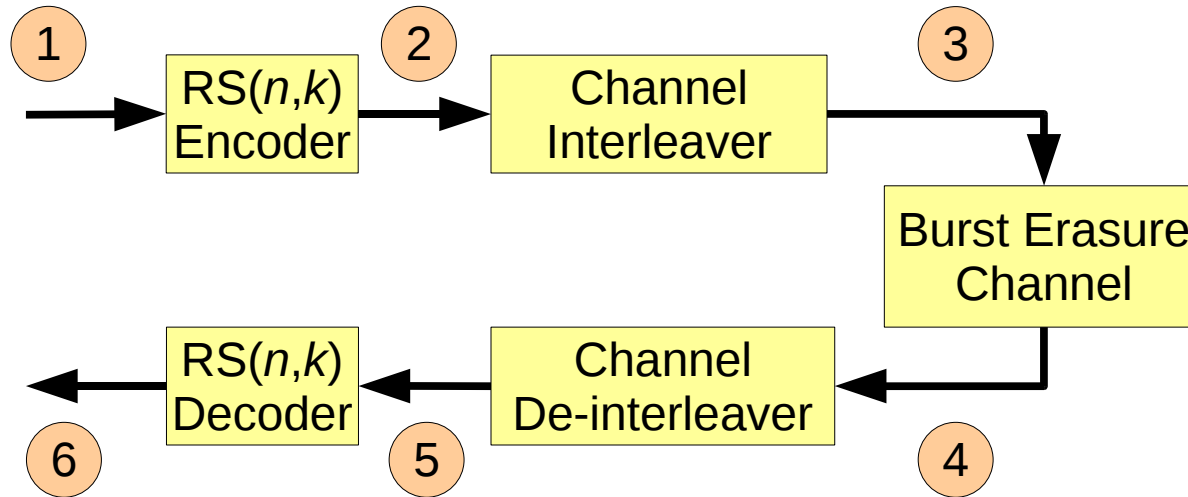


Data flow:

2 RS codewords:



Interleaved RS Codes on Burst Erasure Channel



Data flow:

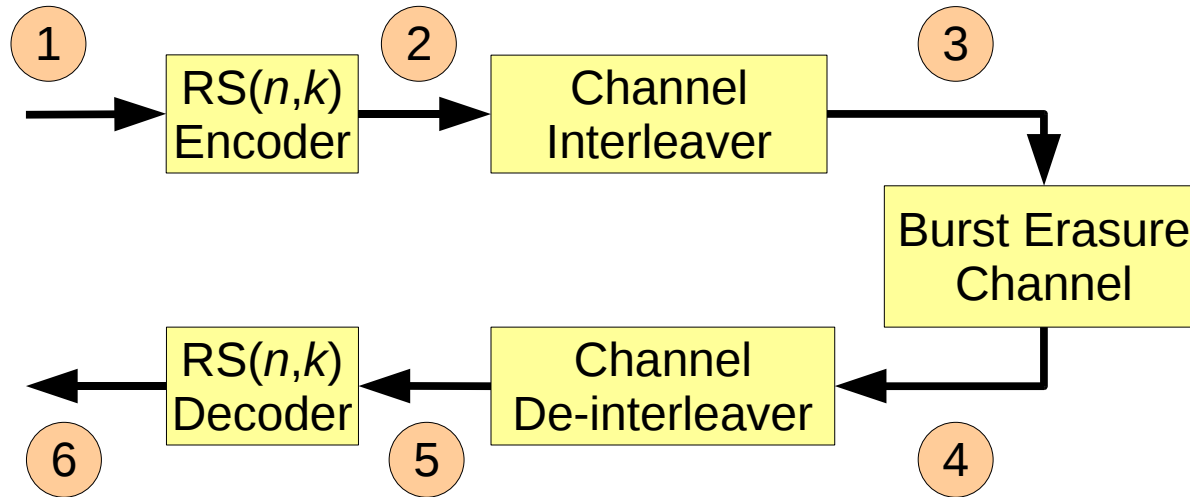
2 RS codewords, written in rows:

1	Data	Parity
2	Data	Parity
3	Data	Parity
	⋮	⋮
<i>l</i>	Data	Parity

3 Interleaver reads column by column, one symbol (J bits) at a time. This is the transmission order:



Interleaved RS Codes on Burst Erasure Channel



Data flow:

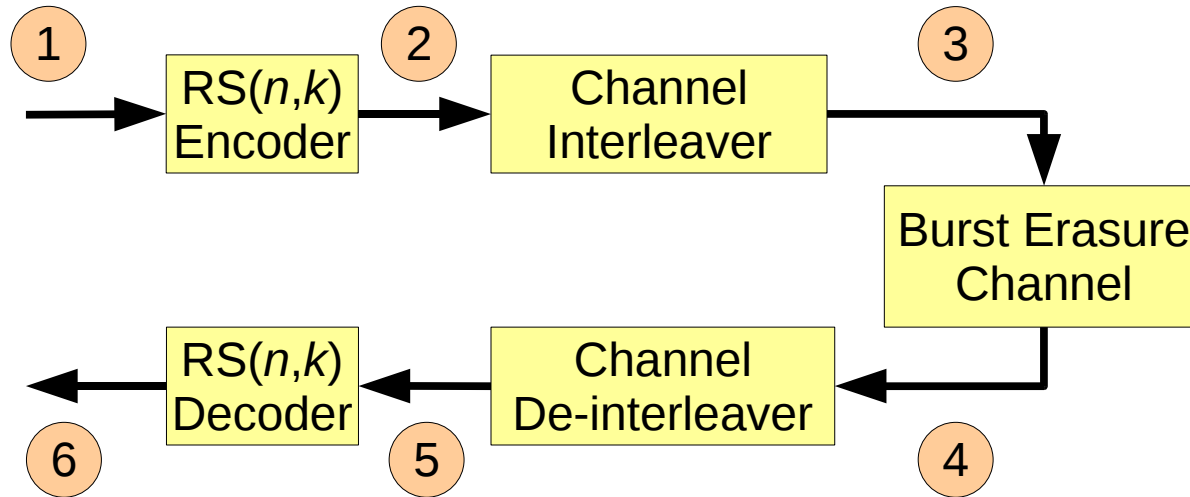
- 4 Burst of erasures knocks out contiguous block of bits (and symbols):

Start of erasure burst



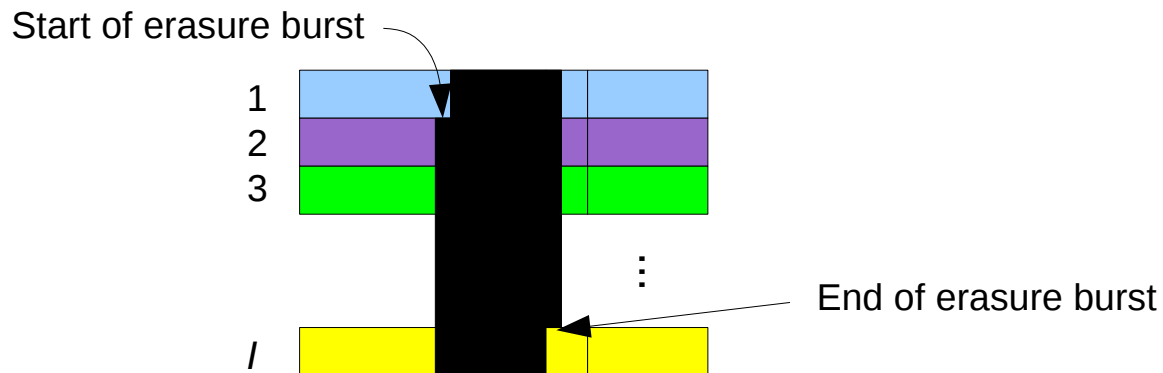
End of erasure burst

Interleaved RS Codes on Burst Erasure Channel

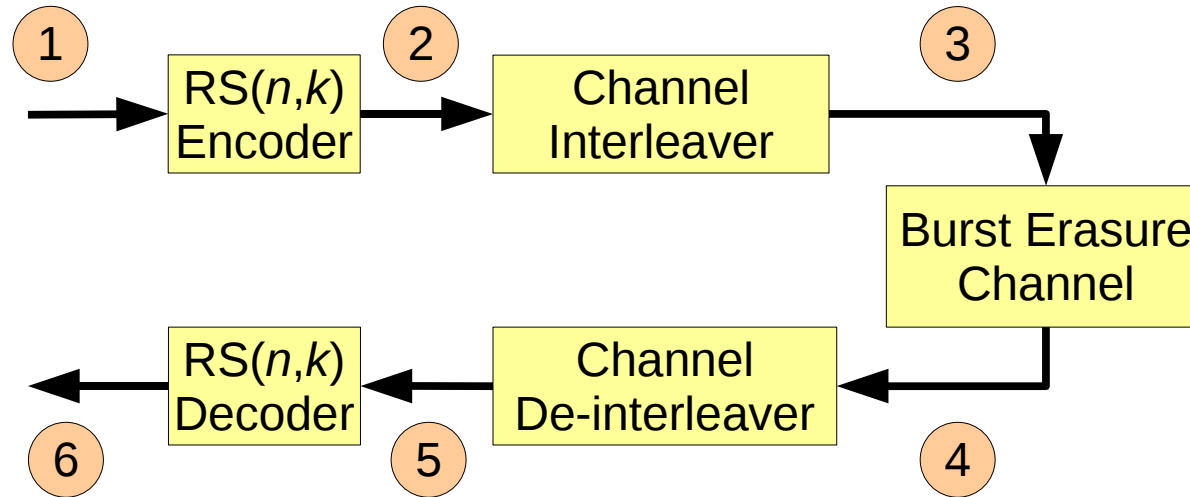


Data flow:

- 5** After de-interleaving, each codeword sees the same number of erasures: (within one)

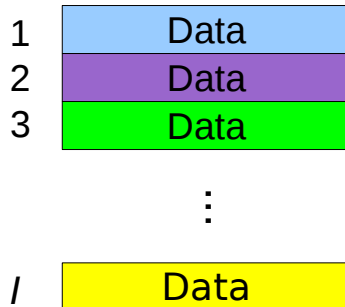


Interleaved RS Codes on Burst Erasure Channel



Data flow:

- 6 RS(n,k) decoder corrects erasures, and returns recovered data:



Performance

Theorem: An (n,k) RS code over $GF(2^J)$ that is block interleaved to depth I has $L_{\max} = (n-k)IJ - J + 1$. Every (nIJ, kIJ) code (same total length and rate) has $L_{\max} \leq (n-k)IJ$.

Sketch of Proof:

An (n,k) RS code is MDS and can correct an code symbol erasure burst of length $n-k$. Each code symbol is J bits, and the interleaver distributes the burst among I codewords, accounting for the additional factor of IJ . (An exact count is done in the recent article by Jon Hamkins.)

The last part of the theorem is the Singleton bound.

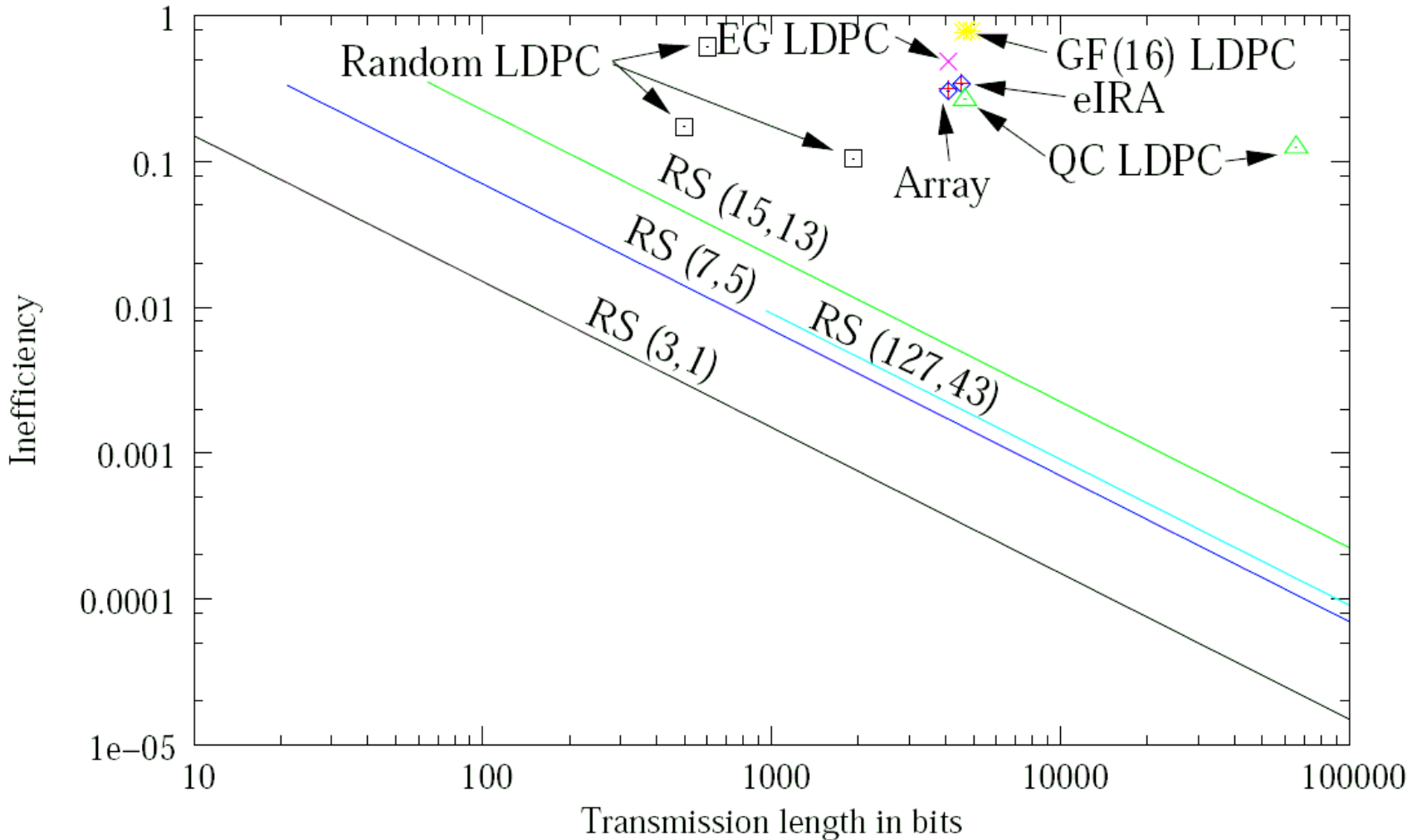
Comments on Performance

- An interleaved RS code of rate r and total transmission length nI has inefficiency

$$\mu = \frac{J - 1}{nIJ(1 - r)}$$

- For fixed n , the inefficiency can be made as close to 0 as desired by increasing I
- This means that very short RS codes can be used with long interleavers, and achieve near-optimal L_{\max}

Example 1: a (3,1) RS code over GF(4), interleaved to total transmission length 1000, achieves $\eta = 0.999$, two 9's more than equivalent rate and length random LDPC codes reported in Sridharan et al. , ISIT 2008



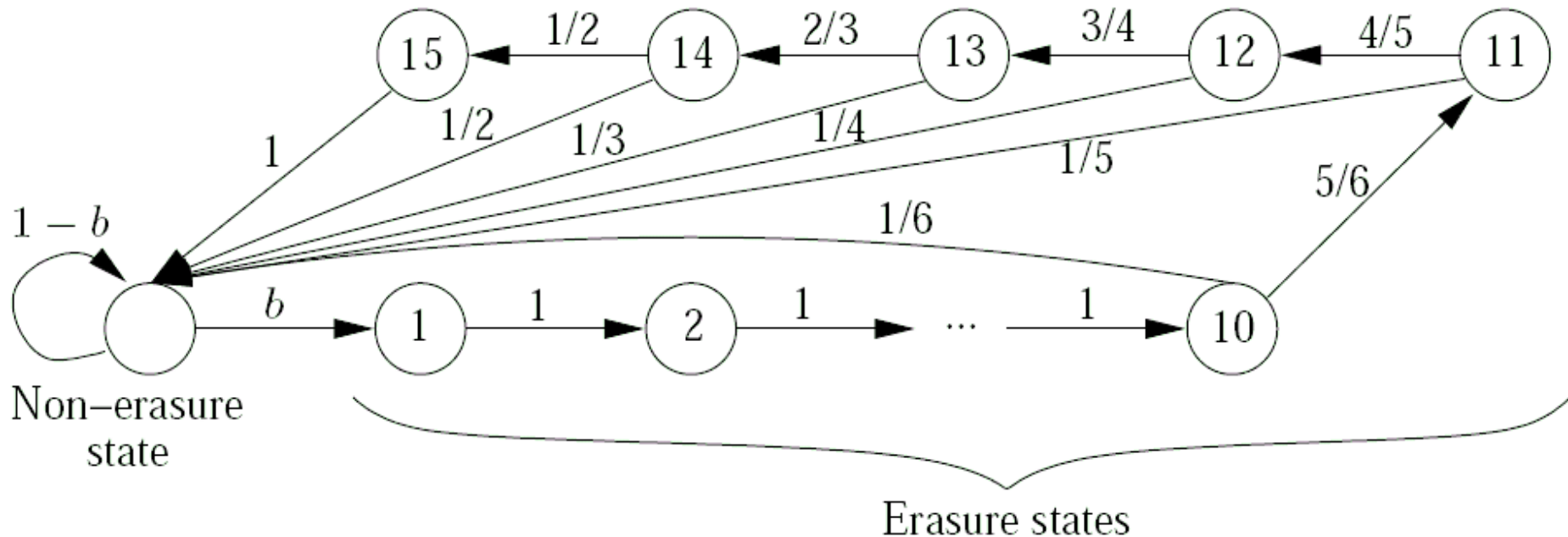
Example 2: Consider the burst erasure channel proposed by Calzolari et al.,
 “Channel coding for future space missions: New requirements and trends.
 Proceedings of the IEEE,” 95(11):2157–2170, Nov. 2007:

8 kbits transmitted per frame;

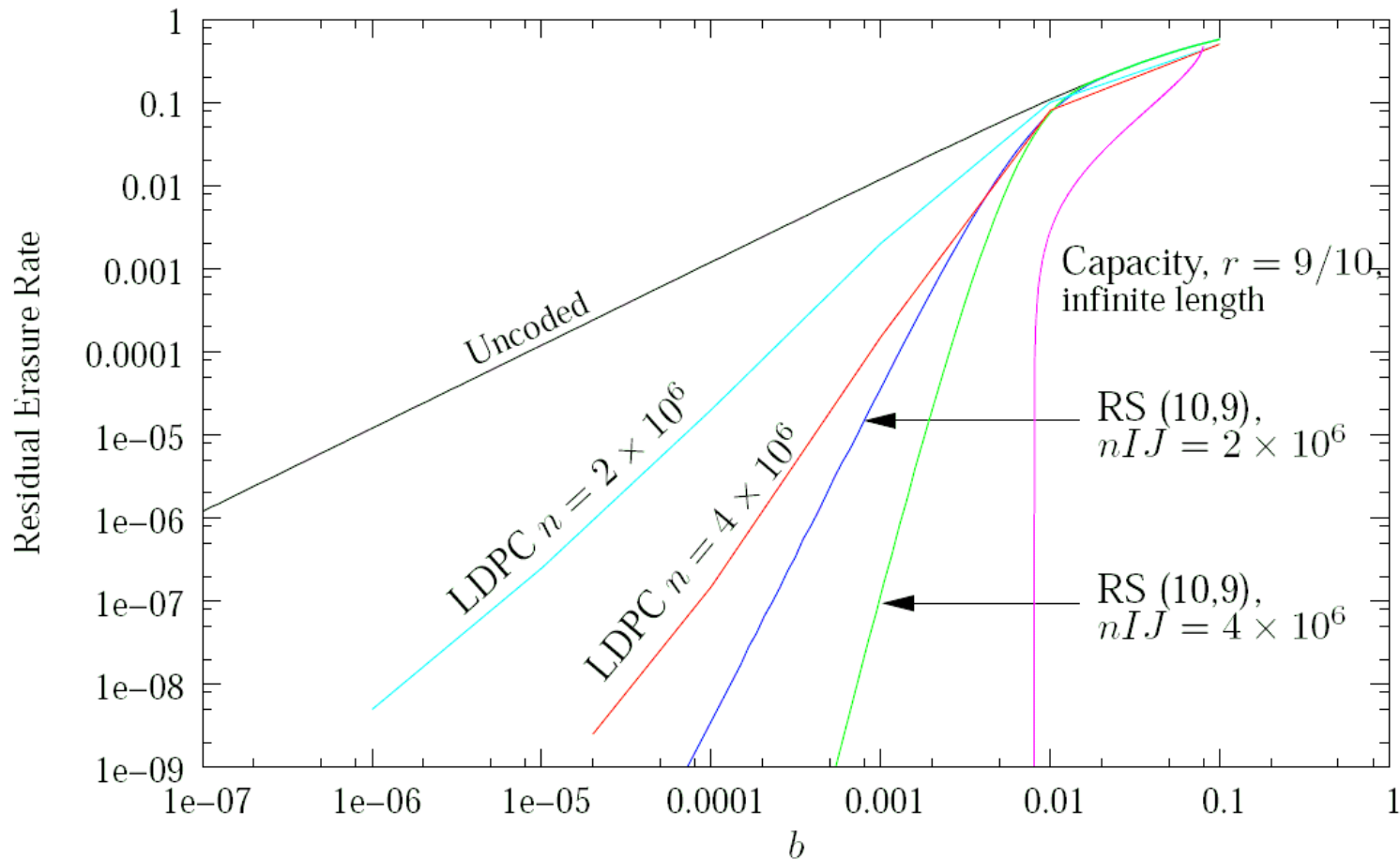
Stay in non-erasure state with probability $1-b$;

Once in bad state, return to good state after B frames, B uniform in $(10,15)$

That is equivalent to this Markov chain:



Example 2 (continued): Comparison of previously reported $r = 9/10$ LDPC codes to block interleaved (10,9) (shortened) RS codes over GF(16). Capacity of rate 9/10 coding is shown.



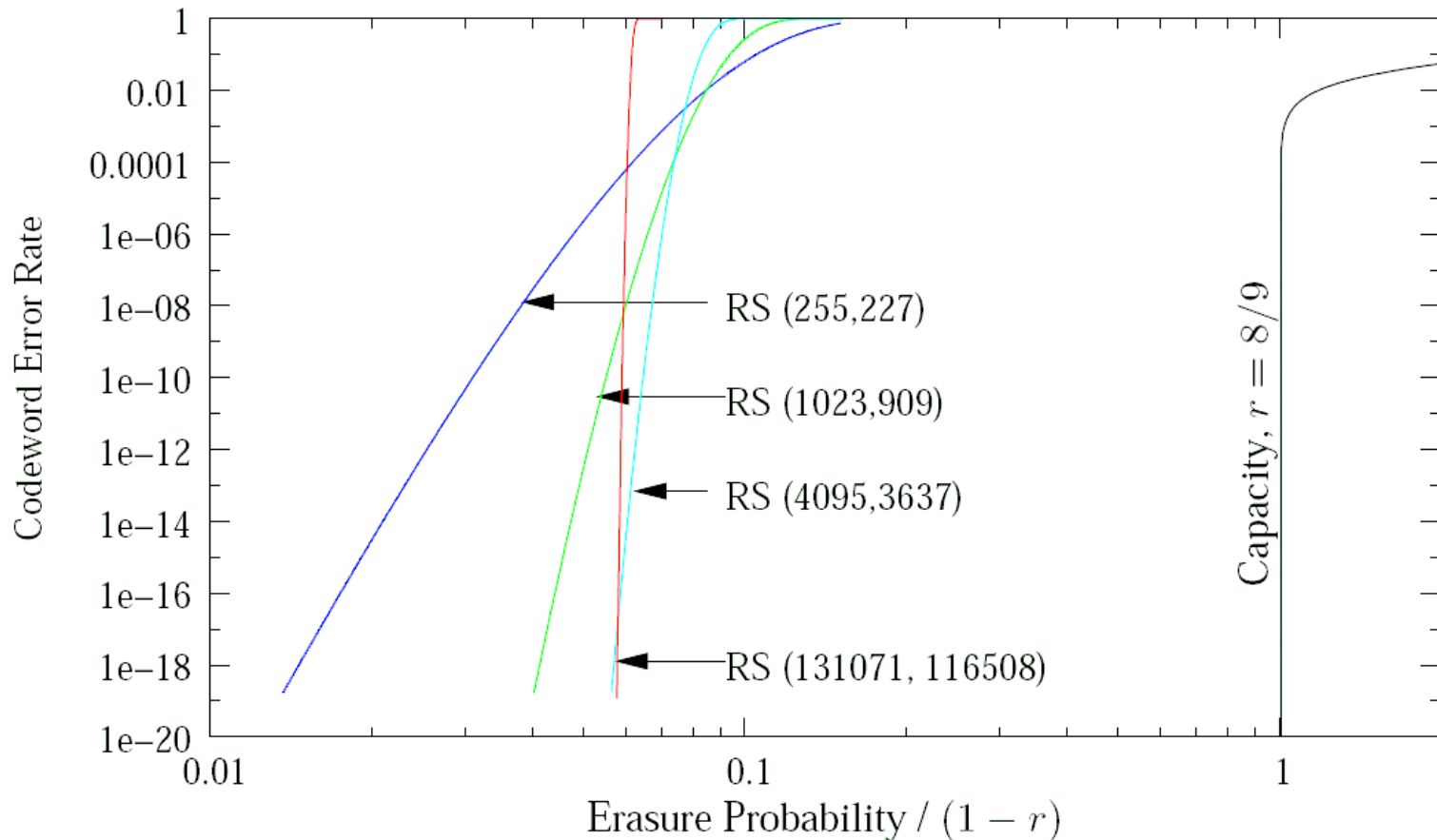
Complexity

Very short RS codes (e.g., (3,2) or (10,9)) have low complexity. For arbitrarily long transmission lengths, the same fixed RS decoder may be used, so the complexity is linear in the transmission length.

LDPC codes generally have super-linear complexity, even for structured quasi-cyclic codes, because more iterations may be required for messages to fully permeate throughout the decoding graph.

Memoryless Erasure Channel

- If channel erasures are memoryless (random and independent), instead of bursty, RS codes are not a good solution because code is over a higher field
- E.g., for rate 8/9 coding, RS codes don't achieve even 10% efficiency
- LDPC codes can achieve ~75% efficiency on burst erasure channels, and are robust to memoryless erasures.



Conclusions

- RS code + channel interleaver is a good solution for burst erasure channel
 - Performance is near-optimal among codes of given rate and transmission length
 - RS achieves efficiency 0.999, with long interleaver
 - LDPC and other codes are more complex and typically have efficiency < 0.9
 - High-speed RS hardware is mature and available
- RS codes are not appropriate for non-bursty (random) erasure channel
 - 1 channel bit erasure results in a whole RS symbol (e.g., 8 bits) erasure
 - This may reduce erasure correction capability by factor of up to J (e.g., 8)